Squark and slepton masses as probes of supersymmetric $SO(10)$ unification

B. Ananthanarayan
Centre for Theoretical Studies,
Indian Institute of Science, Bangalore, India

I. OUTLINE

- **Introduction**
- **$SO(10)$ symmetry breaking and conventional and flipped embedding**
- **Renormalization group equations and sum rules**
- **Sparticle spectrum and discussion**

II. INTRODUCTION

- **Grand unification of the standard model gauge group ($SU(3) \times SU(2) \times U(1)$) into a simple group**
- **Matter and Higgs particles transforming under irreducible representations of this simple group**
- **Supersymmetry the only framework in which the Higgs sector SM is natural**
- **Simple groups like $SU(5)$ or $SO(10)$.**
- **16 can accomodate an entire generation of standard model fermions, and a right-handed neutrino.**
- **The $SO(10)$ framework extended to include supersymmetry.**
- **A complex 10 $SO(10)$ accomodates the two Higgs doublets of minimal supersymmetric standard model (MSSM).**
- **Rank of $SO(10)$ exceeds that of SM by 1.**
- **The breaking of the additional $U(1)$ factor leads to non-universal contributions to the otherwise universal soft SUSY breaking scalar masses of the minimal supersymmetric models (D-term corrections)**

  $m_i^2 = m_0^2 + Y_i^{\perp} g_{10}^2 D$.
- **Consequences at low energies: reach certain regions of parameter space which are not accessible with universal boundary conditions**
- **Non-universality may have a dramatic impact on the sum rules satisfied by the squark and slepton masses.**
- **Consequences to spectrum**

III. CONVENTIONAL AND FLIPPED EMBEDDING OF THE SM IN $SO(10)$

Group theory

$SO(10) \supset SU(5) \times U(1)$

$SO(10) \supset SO(6) \times SO(4)$.

Since $SO(6) \equiv SU(4)$ **and** $SO(4) \equiv SU(2) \times SU(2)$

$SO(10) \supset SU(4) \times SU(2) \times SU(2)$.

16 decomposed under subgroup $SO(10) \supset SU(5) \times U(1)_Z$ **as**

$$\mathbf{16} = \mathbf{5^*}_3 + \mathbf{10}_{-1} + \mathbf{1}_{-5}, \tag{1}$$

$$\mathbf{10} = \mathbf{5}_2 + 5^*{}_{-2}. \tag{2}$$

$SU(5) \supset SU(3)_C \times SU(2)_L \times U(1)_X$, **we have the decomposition**

$$\begin{aligned} \mathbf{5} &= (\mathbf{3},\mathbf{1})_{-2} + (\mathbf{1},\mathbf{2})_3, \\ \mathbf{5}^* &= (\mathbf{3}^*,\mathbf{1})_2 + (\mathbf{1},\mathbf{2})_{-3}, \\ \mathbf{10} &= (\mathbf{3},\mathbf{2})_1 + (\mathbf{3}^*,\mathbf{1})_{-4} + (\mathbf{1},\mathbf{1})_6, \\ \mathbf{1} &= (\mathbf{1},\mathbf{1})_0. \end{aligned} \tag{3}$$

$U(1)_X \subset SU(5)$, **is not identical with the** $U(1)_Y$ **of the SM at this stage.**

Each 16 includes two pairs of $(\mathbf{3}^*, \mathbf{1})$ **and** $(\mathbf{1}, \mathbf{1})$.

The hypercharge $U(1)_Y$ **must be a linear combination of** $U(1)_X$ **and** $U(1)_Z$

1. $Y = X$, **(Georgi-Glashow)**
2. $Y = -\frac{1}{5}(X + 6Z)$, **(flipped)**

upto an overall normalization factor.

1. $Y^\perp = -Z$**(Georgi-Glashow)**
2. $Y^\perp = \frac{-4X+Z}{5}$ **(flipped)**

IV. SOLUTIONS OF RENORMALIZATION GROUP EQUATIONS AND SUM RULES

Squarks and sleptons of the lightest generations RG equations for the soft scalar masses are

$$16\pi^2 \frac{dm^2_{\tilde{Q}_L}}{dt} = -\frac{32}{3} g_3^2 M_3^2 - 6 g_2^2 M_2^2 - \frac{2}{15} g_1^2 M_1^2 + \frac{1}{5} g_1^2 S, \tag{4}$$

$$16\pi^2 \frac{dm^2_{\tilde{u}_R}}{dt} = -\frac{32}{3} g_3^2 M_3^2 - \frac{32}{15} g_1^2 M_1^2 - \frac{4}{5} g_1^2 S, \tag{5}$$

$$16\pi^2 \frac{dm^2_{\tilde{d}_R}}{dt} = -\frac{32}{3} g_3^2 M_3^2 - \frac{8}{15} g_1^2 M_1^2 + \frac{2}{5} g_1^2 S, \tag{6}$$

$$16\pi^2 \frac{dm^2_{\tilde{L}_L}}{dt} = -6 g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 - \frac{3}{5} g_1^2 S, \tag{7}$$

$$16\pi^2 \frac{dm^2_{\tilde{e}_R}}{dt} = -\frac{24}{5} g_1^2 M_1^2 + \frac{6}{5} g_1^2 S, \tag{8}$$

where $t \equiv \ln(Q/Q_0)$, Q_0 some initial large scale; $M_{3,2,1}$ are running gaugino masses, $g_{3,2,1}$ are the gauge couplings, and

$$S \equiv \mathrm{Tr}(Ym^2) = m^2_{H_u} - m^2_{H_d} + \sum_{\text{families}} (m^2_{\tilde{Q}_L} - 2m^2_{\tilde{u}_R} + m^2_{\tilde{d}_R} - m^2_{\tilde{L}_L} + m^2_{\tilde{e}_R}) \,. \tag{9}$$

The $U(1)_Y$ gauge coupling g_1 (and α_1) is taken to be in a GUT normalization. The quantity S evolves according to

$$\frac{dS}{dt} = \frac{66}{5} \frac{\alpha_1}{4\pi} S \tag{10}$$

which has the solution

$$S(t) = S(t_G) \frac{\alpha_1(t)}{\alpha_1(t_G)}. \tag{11}$$

We note that if $S = 0$ at the initial scale (universal case), then the RG evolution implies it is 0 at all scales.

The solution for the RG equations:

$$
\begin{aligned}
m^2_{\tilde{u}_L} &= m^2_{\tilde{Q}_L}(t_G) + C_3 + C_2 + \frac{1}{36}C_1 + (\frac{1}{2} - \frac{2}{3}\sin^2\theta_W)M_Z^2\cos(2\beta) + \frac{1}{5}K, && (12)\\
m^2_{\tilde{d}_L} &= m^2_{\tilde{Q}_L}(t_G) + C_3 + C_2 + \frac{1}{36}C_1 + (-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W)M_Z^2\cos(2\beta) + \frac{1}{5}K, && (13)\\
m^2_{\tilde{u}_R} &= m^2_{\tilde{u}_R}(t_G) + C_3 + \frac{4}{9}C_1 + \frac{2}{3}\sin^2\theta_W M_Z^2\cos(2\beta) - \frac{4}{5}K, && (14)\\
m^2_{\tilde{d}_R} &= m^2_{\tilde{d}_R}(t_G) + C_3 + \frac{1}{9}C_1 - \frac{1}{3}\sin^2\theta_W M_Z^2\cos(2\beta) + \frac{2}{5}K, && (15)\\
m^2_{\tilde{e}_L} &= m^2_{\tilde{L}_L}(t_G) + C_2 + \frac{1}{4}C_1 + (-\frac{1}{2} + \sin^2\theta_W)M_Z^2\cos(2\beta) - \frac{3}{5}K, && (16)\\
m^2_{\tilde{\nu}_L} &= m^2_{\tilde{L}_L}(t_G) + C_2 + \frac{1}{4}C_1 + \frac{1}{2}M_Z^2\cos(2\beta) - \frac{3}{5}K, && (17)\\
m^2_{\tilde{e}_R} &= m^2_{\tilde{e}_R}(t_G) + C_1 - \sin^2\theta_W M_Z^2\cos(2\beta) + \frac{6}{5}K, && (18)
\end{aligned}
$$

where C_1, C_2 and C_3 are

$$
C_i(t) = \frac{a_i}{2\pi^2}\int_t^{t_G} g_i(t)^2 M_i(t)^2,\ i = 1, 2, 3 \tag{19}
$$

$$
a_1 = \frac{3}{5}, a_2 = \frac{3}{4}, a_3 = \frac{4}{3}, \tag{20}
$$

and

$$
K = \frac{1}{16\pi^2}\int_t^{t_G} g_1^2(t)\ S(t)\ dt = \frac{1}{2b_1}S(t)\left[1 - \frac{\alpha_1(t_G)}{\alpha_1(t)}\right], \tag{21}
$$

is the contribution of the non-universality parameter S to the sfermion masses, and $b_1 = 33/5$.

- **RHS of the equations above contain a term proportional to $Y_i(\mathrm{Tr} Y m^2)$.**

- **Find those combinations in which the contribution proportional to K vanishes: $\sum_i \alpha_i m_i^2$, where the α_i are to be determined (i sfermion index)**

$$\sum_i \alpha_i Y_i = 0. \tag{22}$$

- **GUT scale boundary condition: $m_i^2 = m_0^2 + Y_i^\perp g_{10}^2 D$.**

  The universal mass term contribution as well as the D term contribution vanish:

$$\sum_i \alpha_i = 0, \tag{23}$$

$$\sum_i \alpha_i Y_i^\perp = 0. \tag{24}$$

- **Solve these 3 constraint equations for the 5 mass parameters,**

- **Arbitrarily fix coefficient of the right handed selectron in the sum rules to be -1, and ask for integer values for the parameter α_i for the squark doublet.**

- **We then arrive at the following two sum rules:**

$$2m_{\tilde{Q}}^2 - m_{\tilde{u}_R}^2 - m_{\tilde{e}_R}^2 = (C_3 + 2C_2 - \frac{25}{18}C_1), \tag{25}$$

$$m_{\tilde{Q}}^2 + m_{\tilde{d}_R}^2 - m_{\tilde{e}_R}^2 - m_{\tilde{L}}^2 = (2C_3 - \frac{10}{9}C_1), \tag{26}$$

  where we have used the notation

$$m_{\tilde{Q}}^2 = \frac{1}{2}(m_{\tilde{u}_L}^2 + m_{\tilde{d}_L}^2), \;\; m_{\tilde{L}}^2 = \frac{1}{2}(m_{\tilde{e}_L}^2 + m_{\tilde{\nu}_L}^2).$$

- **The essential point is that we have for the conventional embedding $Y = X$ and $Y^\perp = -Z$, and have solved the constraint equations (22) and (24).**

- **Furthermore, the solutions we obtain for flipped embedding is exactly the same!**

- **Flipped embeddding is a special case of $Y = aX + bZ$ and $Y^\perp = cX + dZ$, where a, b, c, d are arbitrary constants for which we get the same solution.**

- **This is a remarkable property of $SO(10)$ unification**

- **No uniqueness in the choice of embedding but gauge symmetry preserves certain relations.**

- **The solution for K is obtained by eliminating C_1, C_2, C_3, m_{16}^2, and m_{10}^2 from the sfermion mass equations. We get for the conventional embedding the result:**

$$K = \frac{1}{4}(m_{\tilde{Q}}^2 - 2m_{\tilde{U}^c}^2 + m_{\tilde{D}^c}^2 + m_{\tilde{E}^c}^2 - m_{\tilde{L}}^2 + \frac{10}{3}\sin^2\theta_W M_Z^2 \cos 2\beta). \quad (27)$$

- Eigenvalues of to $Y^\perp$ of flipped embedding identical to that of conventional.

- **In SO(10) unification, gauge symmetry**

  (i) protects the sum rules irrespective of the embedding;

  (ii) the explicit $D-$ term contributions to the sfermion masses for the two embeddings are in fact identical.

  We note that (ii) implies (i), but not the other way round.

V. CONSEQUENCES TO SPECTRUM

- **Procedure:**
  - **First specify scalar masses at the unification scale (m_0 and $g_{10}^2 D$),**
  - **trilinear couplings (A)**
  - **the Yukawa couplings having a unified value h at the unification scale**
  - **and α_G,**
  - **integrate the set of coupled differential equations down to the effective supersymmetry scale of ~ 1 TeV.**
- **Of the two minimization conditions for the Higgs potential, one can be written as**

$$\frac{\mu_1^2 - \mu_2^2 \tan^2\beta}{\tan^2\beta - 1} = \frac{m_Z^2}{2}. \tag{28}$$

- **Determining $\tan\beta$ from the accurately known value of the τ-lepton mass**
- **inserting it into the equation above, and using the values of the Higgs mass squared parameters determined from the evolution of the parameters yields, from the relations $\mu_1^2 = m_{H_d}^2 + \mu^2$ and $\mu_2^2 = m_{H_u}^2 + \mu^2$, obtain the parameter μ.**
- **Features: Common gaugino mass $M_{1/2}$ are required to ensure that the gluino is sufficiently heavy,**
- **Fairly large values of m_0 ($< M_{1/2}$) are required to ensure that the neutralino is the LSP (in order to prevent the lightest slepton from becoming the LSP).**
- **An upper bound on m_0 ensues when we require a sufficiently large m_A ($m_A = \mu_1^2 + \mu_2^2$). Keeping these features in mind, we study the effects of the $D-$term on the spectrum.**
- **In our work we have chosen the sign of D to be positive.**

- **Results are presented in figures for some typical cases.**
- **Considerable freedom is gained in parameter space that is reachable. The D-term contribution is restricted by requirement that the LSP is neutral.**

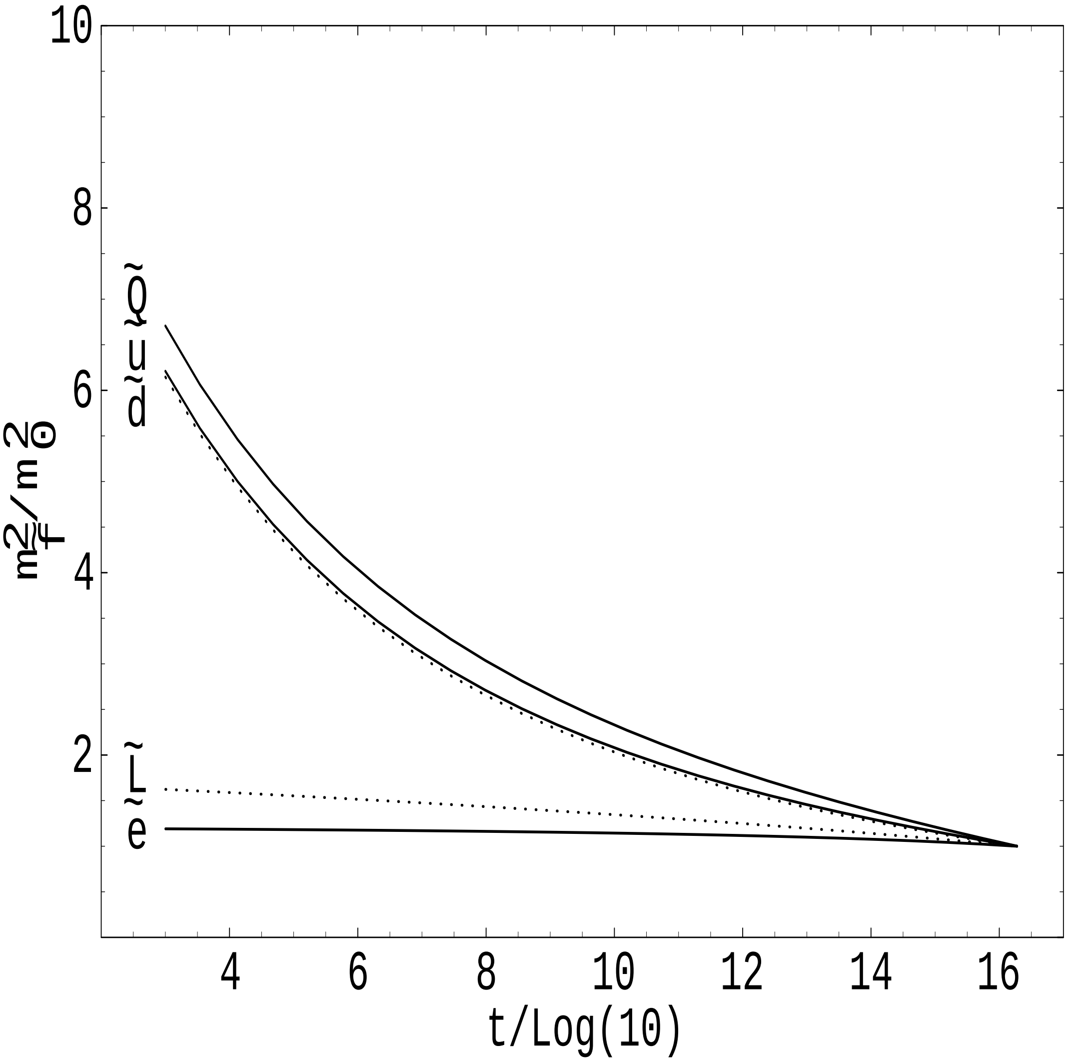

FIG. 1. Evolution of the ratio of the sfermion mass squared parameters to the universal mass squared parameter, as a function of the momentum scale, for the lightest generations. Here we have taken $D = 0$. The values of parameters are $M_{1/2} = 800, m_0 = 700, A_0 = 0$ GeV, $h_t = h_b = h_\tau = 2.0$.

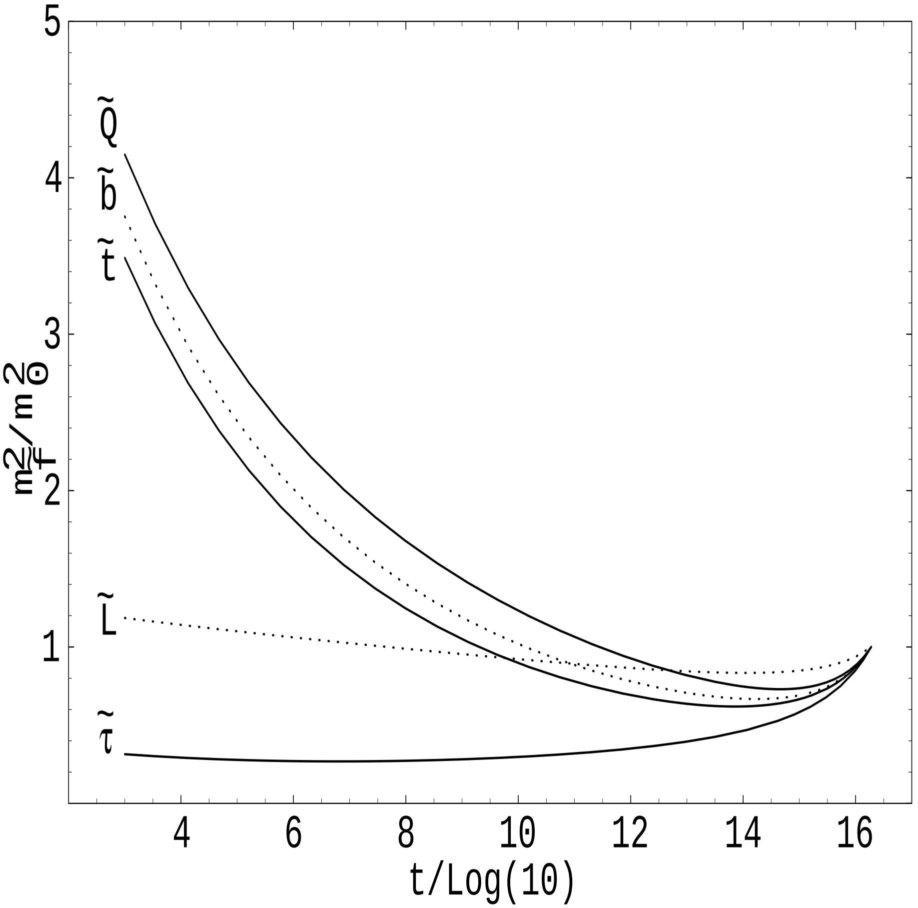

FIG. 2. Evolution of the ratio of the sfermion mass squared parameters to the universal mass squared parameter, as a function of the momentum scale, for the heaviest generation. The value of $D = 0$. The values of parameters are $M_{1/2} = 800, m_0 = 700, A_0 = 0$ GeV, $h_t = h_b = h_\tau = 2.0$.

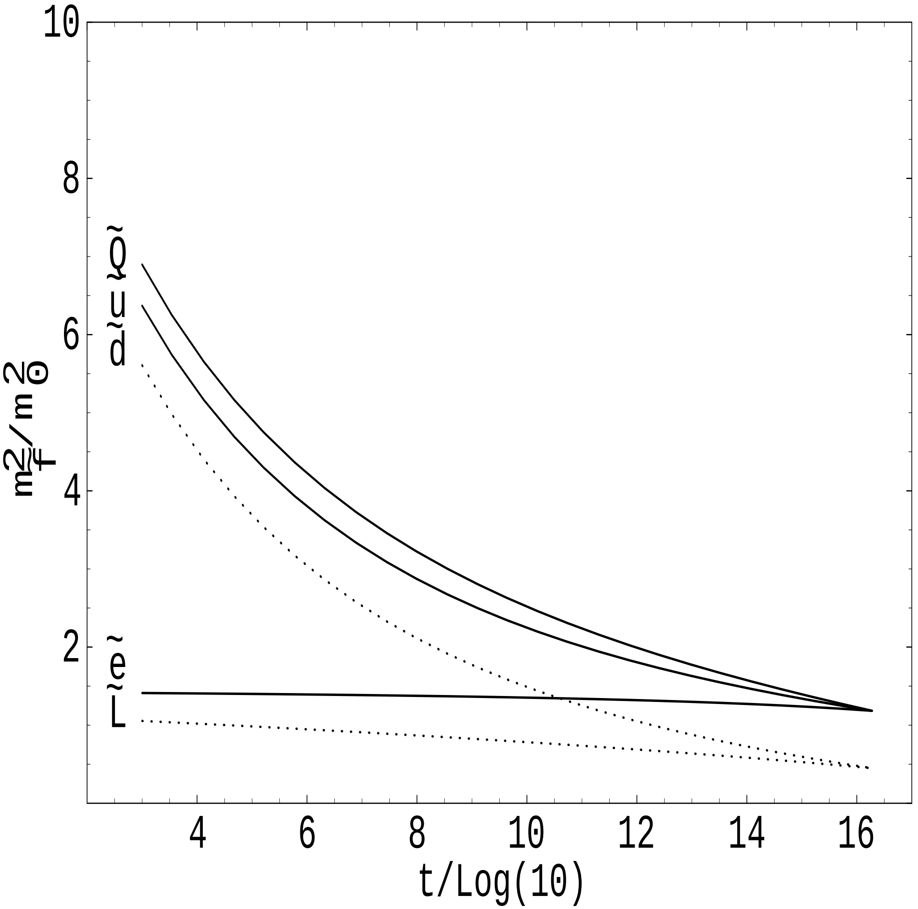

FIG. 3. Evolution of the ratio of the sfermion mass squared parameters to the universal mass squared parameter, as a function of the momentum scale, for the heaviest generation with $g_{10}^2 D = 300$ GeV. The values of parameters are $M_{1/2} = 800, m_0 = 700, A_0 = 0$ GeV, $h_t = h_b = h_\tau = 2.0$.

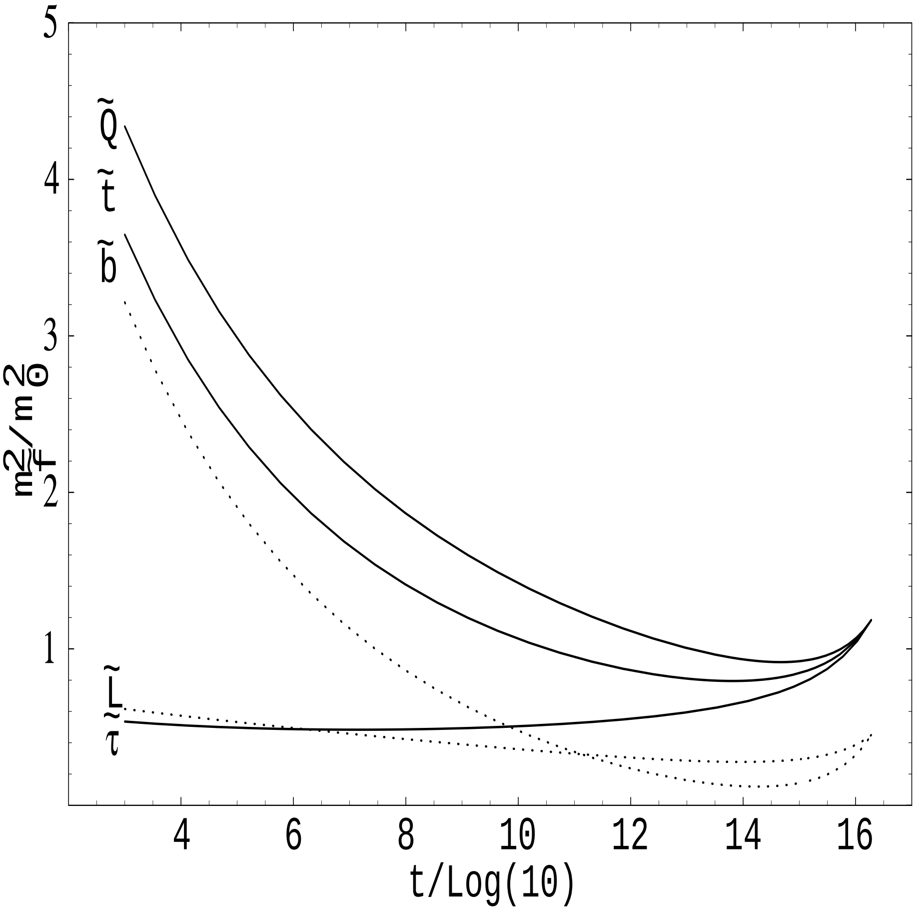

FIG. 4. Evolution of the ratio of the sfermion mass squared parameters to the universal mass squared parameter, as a function of the momentum scale, for the heaviest generation with $g_{10}^2 D = 300$ GeV. The values of parameters are $M_{1/2} = 800, m_0 = 700, A_0 = 0$ GeV, $h_t = h_b = h_\tau = 2.0$.

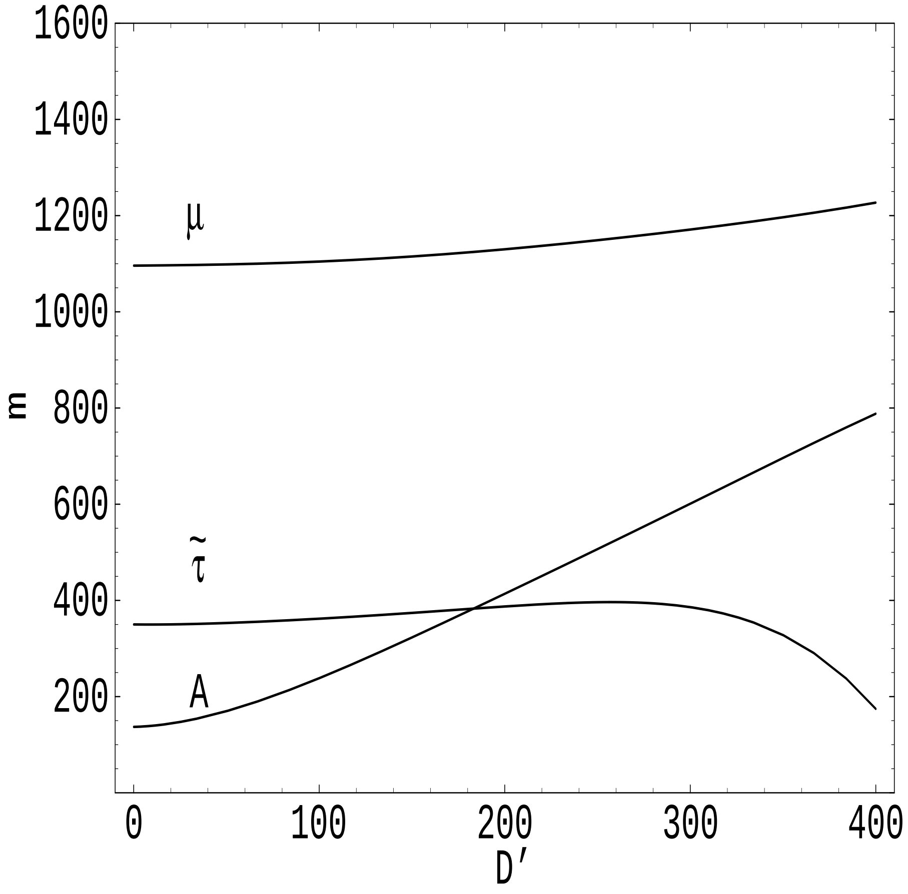

FIG. 5. Values of mass parameters at the supersymmetry breaking scale for the choice of parameters $M_{1/2} = 800$, $m_0 = 700$ $A_0 = 0$ $h_t = h_b = h_\tau = 2.0$, when $D'(\equiv g_{10}^2 D)$ is varied from 0 to 400 GeV.