

Ancient Indian Mathematics : an overview

The aim of this article is to present an overview of ancient Indian mathematics together with a discussion on the sources and directions for future studies. For this purpose it would be convenient to divide the material broadly into the following groups:

1. Vedic mathematics
2. Mathematics from the Jaina tradition
3. Development of the number system and numerals
4. The mathematical astronomy tradition
5. Paṭīgaṇita and the Bakhshālī manuscript
6. The Kerala school of Mādhava

We shall discuss these topics individually with the points as above in view.

Vedic mathematics

A major part of the body of mathematical knowledge from the Vedic period that has come down to us is from the *Śulvasūtras*. The *Śulvasūtras* are compositions aimed at providing instruction on the principles involved and procedures of construction of the *vedis* (altars) and *agnis* (fireplaces) for the performance of the *yajnas*, which were a key feature of the Vedic culture. The fireplaces were constructed in a variety of shapes such as falcons, tortoise, chariot wheels, circular trough with a handle, pyre, etc (depending on the context and purpose of the particular *yajna*) with sizes of the order of 20 to 25 feet in length and width, and there is a component of the *Śulvasūtras* describing the setting up of such platforms with tiles of moderate sizes, of simple shapes like squares, triangles, and occasionally special ones like pentagons. Many of the *vedis* involved, especially for the *yajnas* for special occasions had dimensions of the order of 50 to 100 feet, and making the overall plan involved being able to draw perpendiculars in that setting. This was accomplished both through the method that is now taught in schools, involving perpendicularity of the line joining the centres of two intersecting circles with the

line joining the two points of intersection, as also via the use of the converse of “Pythagoras theorem”; they were familiar with the “Pythagoras theorem”, and explicit statement of the theorem is found in all the four major *Śulvasūtras*. The *Śulvasūtras* also contain descriptions of various geometric principles and constructions, including procedures for converting a square into a circle with equal area, and vice versa, and a good approximation to the square root of 2 (see [4] for some details).

The *Śulvasūtras*, like other Vedic knowledge, were transmitted only orally over a long period. There have also been commentaries on some of the *Śulvasūtras* in Sanskrit, but their period remains uncertain. When the first written versions of the *Śulvasūtras* came up is unclear. The text versions with modern commentaries were brought out by European scholars (Thibaut, Bürk, van Gelder and others) starting from the second half of the nineteenth century (see [42], [27], [28], [37], [21], [6], [44]). With regard to genesis of his study of the *Śulvasūtras* Thibaut mentions that the first to direct attention to the importance of the *Śulvasūtras* was Mr. A.C. Burnell, who in his *Catalogue of a Collection of Sanscrit Manuscripts*, p 29, remarks that “we must look to the *Śulva* portions of the *Kalpasūtras* for the earliest beginnings among the Brāhmaṇas.”.

While the current translations are reasonably complete, some parts have eluded the translators, especially in the case of *Mānava Śulvasūtra* which turns out to be more terse than the others. New results have been brought to light by R.G. Gupta [13], Takao Hayashi and the present author (see [4], § 3), and perhaps also by others, not recognised by the original translators. Lack of adequate mathematical background on the part of the translators could be one of the factors in this respect. There is a case for a relook on a substantial scale to put the mathematical knowledge in the *Śulvasūtras* on a comprehensive footing. There is also scope for work in the nature of interrelating in a cohesive manner the results described in the various *Śulvasūtras*. The ritual context of the *Śulvasūtras* lends itself also to the issue of interrelating the ritual and mathematical aspects, and correlating with other similar situations from other cultures; for a perspective on this the reader may refer Seidenberg [36].

Another natural question that suggests itself in the context of the *Śulvasūtras* is whether there are any of the fireplaces from the old times to be found. From the description of the brick construction it would seem that they would have been too fragile to withstand the elements for long; it should be borne in mind that

the purpose involved did not warrant a long-lasting construction. Nevertheless, excavations at an archaeological site at Singhol in Panjab have revealed one large brick platform in the traditional shape of a bird with outstretched wings, dated to be from the second century BCE ([18], p.79-80 and [25], footnote on page 18); it however differs markedly from the numerical specifications described in the *Śulvasūtras*. This leaves open the possibility of finding other sites, though presumably not a very promising one.

Apart from the *Śulvasūtras*, mathematical studies have also been carried out in respect of the Vedas, mainly concerning understanding of the numbers. For a composition with a broad scope, including spiritual and secular, the *Rgveda* shows considerable preoccupation with numbers, with numbers upto 10,000 occurring, and the decimal representation of numbers is seen to be rooted there; see [2]. (It should be borne in mind however that the numbers were not written down, and the reference here is mainly to number names.) The *Yajurveda* introduces names for powers of 10 upto 10^{12} and various simple properties of numbers are seen to be involved in various contexts; [25] for instance. There is scope for further work in understanding the development as a whole; this would involve familiarity with mathematics on the one hand and knowledge of Vedic sanskrit on the other hand.

Mathematics from the Jaina tradition

There has been a long tradition among the Jainas of engaging with mathematics. Their motivation came not from any rituals, which they abhorred, but from contemplation of the cosmos, of which they had evolved an elaborate conception. In the *Jaina* cosmography the universe is supposed to be a flat plane with concentric annular regions surrounding an innermost circular region with a diameter of 100000 *yojanas* known as the *Jambudvīpa* (island of Jambu), and the annular regions alternately consist of water and land, and their widths increasing twofold with each successive ring; it may be mentioned that this cosmography is also found in the *Purānas*. The geometry of the circle played an important role in the overall discourse, even when the scholars engaged in it were primarily philosophers rather than practitioners of mathematics. Many properties of the circle have been described in *Sūryaprajñāpti* which is supposed to be from the fourth or fifth century BCE (earliest extant manuscript is from around 1500, on paper) and in the work of Umāsvāti, who is supposed to have lived around 150 BCE according to the *Śvetāmbara* tradition and in the second century CE according to the *Digambara* tradition of Jainas.

One of the notable features of the Jaina tradition is the departure from old belief of 3 as the ratio of the circumference to the diameter; *Sūryaprajñāpti* recalls the then traditional value 3 for it, and discards it in favour of $\sqrt{10}$. The *Jainas* were also aware from the early times that the ratio of the area of the circle to the square of its radius is the same as the ratio of the circumference to the diameter. They had also interesting approximate formulae for the lengths of circular arcs and the areas subtended by them together with the corresponding chord. How they arrived at these formula is not understood. Permutations and combinations, sequences, categorisation of infinities are some of the other mathematical topics on which elaborate discussion is found in Jaina literature.

Pronounced mathematical activity in the Jaina tradition is seen again from the 8th century, and it may have continued until the middle of the 14 th century. *Gaṇita-sāra-saṅgraha* of Mahāvīra, written in 850, is one of the well-known works in this respect. Vīrasena (8th century), Śrīdhara (between 850 and 950), Nemīcandra (around 980 CE), Ṭhakkura Pherū (14 th century) are some of the other names that may be mentioned with regard to development of mathematics in the Jaina canon.

An approximation for π was given by Vīrasena by: “sixteen times the diameter, together with 16, divided by 113 and thrice the diameter becomes a fine value (of the circumference)”. There is something strange about the formula that it prescribes “together with 16” - surely it should have been known to the author that the circumference is proportional to the diameter and that adding 16, irrespective of the size of the diameter, would not be consistent with this. If one ignores that part (on what ground?) we get the value of π as $3 + \frac{16}{113} = \frac{355}{113}$, which is indeed a good approximation, as the author stresses with the phrase “a fine value (*sūkshmadapī sūkshamam*)”, accurate to seven significant places. The same formula was given by Chong-Zhi in China in the 5th century, (and I was told by a Jaina scholar that the latter also involved the same error mentioned above). This suggests the issue of exploring the mathematical contact with China and the channels through which it may have occurred if it did. Specifically how such a value may have been found (wherever it was found independently) would also be worth exploring from a mathematical point of view.

In the work of Ṭhakkura Pherū from the early 14 th century one sees a combination of the native Jaina tradition together with Indo-Persian literature. Some of

the geometry discussed, involving domes, arches etc., has close connections with the development of Islamic architecture in India. A Jaina astronomer Mahendra Sūri who was at the court of a monarch of the Tughluq dynasty during the late 14th century wrote on the astrolabe. Various links between Sanskrit and Islamic science in Jaina astronomical works have been discussed in [26]

The text of *Gaṇita-sāra-saṃgraha* with English translation was brought out by Rangacharya in 1912, and has been recently reprinted [31]. More recently an edition with English and Kannada translations together with the original text has been brought out by Padmavathamma [22]. An edition of Ṭhakkura Pherū's *Gaṇita-sāra-kaumudī* has been brought out very recently through a collaborative effort [34]. A collection of papers giving an overview of various aspects of Jaina mathematics may be found in [14]; see also [5], an older exposition on the topic. On the whole however there has not been adequate systematic study of the Jaina works from a mathematical point of view (even from a comparative point of view within the context of studies in ancient Indian mathematics). With regard to the older Jaina works, from BCE and the early centuries of CE, there is considerable lack of understanding. Adequate information needs to be pooled up on available resources, in the first place.

Development of the number system and numerals

Study of development of the number system in India cuts across the Vedic, Jaina and Buddhist traditions. From the early times one sees a fascination for large numbers in India, as we noted in the earlier sections on the Vedic and Jaina traditions. Large numbers are also found in the Buddhist tradition, and Buddha himself was renowned for his prowess with numbers; Tallakṣhaṇa, a term from the Buddhist tradition, represented 10^{53} . The names of powers of 10 however differed over traditions and also over period; e.g. Parārdha, which literally means “half-way to heaven”, meant 10^{12} in early literature, it stood for 10^{17} in later works such as of Bhāskara II. The oral tradition of usage of several powers of 10 is likely to have played a crucial role in the emergence of decimal representation in written form at later stage, apparently in the early centuries of the common era. This connection is however not very straightforward; there was a long period, of several hundred years, in between when written form of numbers did not follow the place value notation (see below); besides, even after the decimal place value system with zero came into vogue the other systems seem to have continued to be used for quite a while. One may wonder about the reasons for this in the context of

the frequent references to the powers of 10 in the oral tradition, and the apparent convenience and elegance of the decimal place value system. On the other hand the Chinese seem to have used decimal place value system for representation of numbers, without a symbol for zero in place of which they left a blank space, from very early times and at least from the 3rd century BCE. Introduction of zero as a place holder paved the way for the writing the numbers as we do now, as far as the whole numbers are concerned; the full decimal representation system as we now use, extending also to the fractional part, with a separating decimal point, had its beginnings in the 15 th century Europe, though it is noted to have been first used by Arabs in the 10 th century. Conceptualisation of zero as a number, integrated into the number system, happened in the early centuries of the common era in India, and in Brahmagupta's work *Brāhmasphuṭasiddhānta* we find a systematic exposition, in the seventh century, which includes also arithmetic with negative numbers.

The development of the numerals is a parallel topic. Written numerals in various forms have been studied. The earliest of these could go back to the Indus seals, with strokes representing numbers. Kharoṣṭhi numerals which were used between 3rd century BCE to 3rd century CE and are found in the inscriptions from Kalderra, Takṣaśila and Lorian, and Brāhmi numerals from Nāṇeghat (first century BCE) are some of the ancient numerals; incidentally they did not use the place value system. The earliest extant inscriptions involving the decimal numeral system is said to be from Gujarat, dated 595 CE; it has however been argued by R.Saloman in [32] that this is a spurious inscription. The oldest known zero in an inscription in India is from 876 CE and is found in a temple in Gwalior (an image of this may be viewed online, thanks to Bill Casselman). Much research was done by Bhagwanlal Indraji in the late nineteenth century, an account of which may be found in the book of George Ifrah [16]; (a good deal of what Ifrah says has been contradicted by various reviewers - see [8] for details - one may nevertheless suppose that what he reports from the work of Bhagwanlal Indraji would be reliable).

Apart from the inscriptions in stone, copper plates that were legal documents from around the 7 th to 10th centuries, recording grants of gifts by kings or rich persons to Brāhmanas, have been examined for numerals presented in decimal system. There have been some objections to this source, as the plates are susceptible to forgery, on account of attempts to misappropriate the properties involved,

but while this may apply to a few plates, as a whole the plates may not be discounted as a source; see [7], pages 44-48, for a discussion on this, where the author tries to rebut the objections. Numerals appearing in ancient manuscripts are another source in this respect. Mathematical works in all the traditions involve large numbers and the way the numbers are represented in the extant manuscripts from different times would be an interesting aspect of study. It may be observed that various Indian languages have their own symbols for the individual digits, and the genesis of these systems would also be a related issue. The author is not aware of any comprehensive study on the topic.

A systematic archiving of the material in this respect from various sources is very much called for, followed by an analysis of the path of development of ideas as may be discerned from the sources.

The mathematical astronomy tradition

The Siddhānta or mathematical astronomy tradition has been the dominant stream of mathematics in India, with an essentially continuous tradition that flourished for close to a thousand years, starting from about the third or fourth centuries. Āryabhaṭa (476 - 550) is the first major figure from the tradition and is regarded as the founder of scientific astronomy in India. The Siddhānta tradition indeed continued until Bhāskara II (1114 - 1185), and also beyond, though he is viewed as the last major exponent in the continuity.

The Āryabhaṭīya, written in 499, is the earliest completely surviving composition from among the Siddhānta works and is basic to the tradition, and also to the later works of the Kerala school of Mādhava which I shall discuss below. It consists of 121 verses divided into four chapters Gītikapāda, Ganitapāda, Kalakriyapāda and Golapāda. The first one sets out the cosmology and contains also a table of 24 sine differences at intervals of 225 minutes of arc, in a single verse. The second chapter, as the name suggests, is devoted to mathematics, and includes in particular procedures for finding square roots and cube roots, an approximate expression for π , formulae for areas and volumes of various geometric figures, formulae for sums of consecutive integers, sums of squares, sums of cubes and computation of interest; see [40] and [41] for various details. The other two chapters are concerned with astronomy, dealing with distances and relative motions of planets, eclipses etc. (we shall not go into the details here).

Varāhamihira (505 - 587), Bhāskara I (600 - 680), Brahmagupta (598 - 668), Govindaswamī (800 - 860), Śankaranārāyaṇa (840 - 900), Āryabhaṭa II (920 -

1000), Vijayānandī (940 - 1010), Śrīpati (1019 - 1066), Brahmadeva (1060 - 1130) were some of the major figures during the period until Bhāskara II; Nārāyaṇa Paṇḍit and Gaṇeśa may be named from the later centuries (14 th and 16 th) as directly from the tradition. It should be mentioned that many of the dates quoted here are approximate, as there is no reliable historical information available on them and the dating is based on various indirect inferences.

The bulk of the work presented in Brahmagupta's *Brāhmasphuṭasiddhānta* is on astronomy. There are however two chapters, 12th and 18th, devoted to general mathematics. Also, the 21st chapter has verses dealing with trigonometry, which in Siddhanta astronomy literature used to be combined with astronomy, rather than the mathematical topics discussed in the works. Another special feature of the work is chapter 11, which is a critique on earlier works including *Āryabhaṭīya*; like in other scientific communities this tradition had also many internal controversies; the strong language used would however seem disconcerting by contemporary standards. Chapter 12 is known for a systematic formulation of arithmetic operations, including with negative numbers, which eluded European mathematics until the middle of the second millennium. The chapter also contains geometry, including in particular his famous formula for the area of a quadrilateral generalizing Heron's formula for the area of a triangle; it is however stated without the condition of cyclicity of the quadrilateral that is needed for its validity - a point criticised by later mathematicians in the tradition. The 18th chapter is devoted to the *kuṭṭaka* and other methods for solving second-degree indeterminate equations. The reader is referred to [39] for the original text, and [25], for instance, for a summary of the contents. The *Brāhmsphuṭasiddhānta* considerably influenced mathematics in the Arab world.

Bhāskara II is the author of the famous mathematical texts *Līlāvati* and *Bījagaṇita*. Apart from being an accomplished mathematician he was a great teacher and populariser of mathematics. *Līlāvati*, which literally means "playful", presents mathematics in a playful way, with several verses directly addressing a pretty young woman, and examples presented with reference to various animals, trees, ornaments, etc. (Legend has it that the book is named after his daughter after her wedding failed to materialise on account of an accident with the clock, but there is no historical evidence to that effect.). The book presents apart from various introductory aspects of arithmetic, geometry of triangles and quadrilaterals, examples of applications of the Pythagoras theorem, *trirāsika* and *kuṭṭaka*

methods, problems on permutations and combinations etc. The *Bījagaṇita* is at an advanced level. It is a treatise on Algebra, the first known independent work of its kind in Indian tradition. Operations with unknowns, *kuṭṭaka* and *cakravāla* methods for solutions of indeterminate equations are some of the topics discussed, together with examples. Bhāskara's works on astronomy, *Siddhānta śiromaṇi* and *Karaṇa kutūhala*, contain several important results in trigonometry, and also some ideas of calculus; see [43], a nice recent account of trigonometry from ancient times in various cultures.

The works in the Siddhānta tradition have been edited and there are various commentaries, including many from the earlier centuries, and works by European authors such as Colebrook [3], and many Indian authors including Sudhākara Dvivedī, Kuppaṇṇa Śāstri and K.V. Sarma; see [20] for an overview of the works. The 2-volume book of Datta and Singh [7] and the books of Saraswati Amma [33] and A.K. Bag [1] serve as convenient references for many results known in this tradition. Various details have been described in the book of Plofker [25]; see also [24]. Nevertheless, more detailed accounts suitable for a modern context are called for, in the context of the scope of the subject both from mathematical and historical point of view.

Cataloguing of the extant manuscripts and making them accessible is also a necessary task in this respect. A census of the Sanskrit manuscripts on exact sciences was produced by David Pingree during 1970-94; the role and proportion of mathematics in the body as whole seems to be difficult to ascertain and calls for serious study; a good beginning may be said to have been made in [20], where it is noted in particular that only a small proportion of the listed manuscripts from the census pertain directly to mathematics - the census encompasses all manuscripts with some connection with *jyotiṣa*. A similar compendium on the Sanskrit texts in the repositories in Kerala and Tamilnadu was produced by K.V. Sarma in 2002. A bibliography of Sanskrit works on astronomy and mathematics was also brought out by S.N. Sen, A.K. Bag and R.S. Sarma in 1966 [38].

The related theme that needs to be pursued is to understand the instruments used, such as the clocks, astrolabes and other instruments involved in astronomy and the mathematics underlying them. Considerable work has been done in this respect by S.R. Sarma. Interplay between observational and mathematical aspects is another theme on which work needs been done, it being generally believed that mathematical astronomy overshadowed the study of astronomy in India.

Patīgaṇita and the Bakhshālī manuscript

While a majority of the works and authors dealing with mathematics concerned themselves with astronomy, there have indeed been works dealing exclusively with arithmetic. The term *pātīgaṇita* seems to have come into use for this in Sanskrit works from around the 7th century. Some later authors referred to it as *vyaktaṇita* (calculation with the “known”) contrasting it with *avyaktaṇita* (calculation with the “unknown”, which referred to algebra). Śrīdhara’s *Triśatikā* (ca. 750), Mahāvira’s *Gaṇita-sāra-saṃgraha* (ca. 850), *Gaṇita-tilaka* (1039) of Śrīpati, *Līlāvati* (1150) of Bhāskara, *Gaṇita-kaumudī* (1356) of Nārayaṇa Paṇḍita and some of the major works of this genre. While individually they have many special features in terms of a variety of detail, there is also an underlying thread of unity in presenting arithmetic. There is also another rather unique manuscript which broadly falls in this general category of *patīgaṇita*, viz. the Bakhshālī manuscript, which has been a crucial but enigmatic source in the study of ancient Indian mathematics, with many open issues and controversies around it.

The manuscript was found in 1881, buried in a field in the village Bakhshali, near Peshawar, from which it derives its name. It was acquired by the Indologist A.F.R. Hoernle who studied and published a short account on it, and later in 1902 presented the manuscript to the Bodleian library at Oxford, where it has been since then. The manuscript consists of 70 folios of *bhūrjapatra* (birch bark); of these 51 folios retained a fair proportion of the original - of the rest one folio is blank while others are either much damaged or blank. Birch bark (unlike palm leaf which is another material that was extensively used for manuscripts) is generally known to be a rather fragile material that tends to deteriorate relatively fast and is vulnerable to crumbling on being handled, when it is more than two or three hundred years old; see [45] for a discussion on ancient manuscripts. Fortunately the manuscript was in a condition suitable enough for the early studies, but unluckily certain steps taken for preservation of the leaves are said have apparently made the folios stick together and it is now difficult to separate them (orally received information - needs confirmation). Facsimile copies of all the folios were brought out by Kaye in 1927 [19], which has since then been the source for the subsequent studies. The date of the manuscript has been a subject of much controversy since the early years. Hoernle dated it to be from the 3rd or 4th century while Kaye argued it to be from the twelfth century. Various dates have since then been proposed by many subsequent authors, ranging from the early centuries of

CE to the 10th century. The mathematical contents of the manuscript may be expected to pre-date the manuscript itself and there have been many suggestions in that respect as well. T. Hayashi who produced what is perhaps the most comprehensive account [15] so far, examining various issues in detail, concludes that the manuscript may be assigned some time between the 8th and the 12th century, while the work may most probably be from the 7th century. One way of settling the issue of the date of the manuscript would of course be to use carbon dating techniques. Though some attempts were made by Bill Casselman towards getting this done, they have not borne fruit. One hopes that eventually this will be done, and it may clarify the historical issues around the manuscript.

An approximate formula for extraction of square-roots of non-square numbers, used systematically in many problems in the manuscript, dealing with quadratic equations, has attracted much attention. Some calculations in the manuscript involve computations with fractions with large numerator and denominator (each expressed in decimal representation). One of the verifications of a solution of a quadratic equation involves a fraction whose numerator has 23 digits and the denominator has 19 digits!

The Kerala school of mathematics

In the 1830s Charles Whish, an English civil servant in the Madras establishment of the East India Company, brought to light a collection of manuscripts from a mathematical school that flourished in the central part of Kerala, between Kozikode and Kochi, starting from late fourteenth century and continuing at least into the beginning of the seventeenth century. The school is seen to have originated with Mādhava, to whom his successors have attributed many results presented in their texts. Since the middle of the 20th century Indian scholars have worked on these manuscripts and the contents of the manuscripts have been studied. Apart from Mādhava, Nīlakantha Somayāji was another leading personality from the School. There are no extant works of Mādhava on mathematics, (though some work on astronomy is known). Nīlakantha authored a book called *Tantrasamgraha* (in Sanskrit) in 1500 CE. There was a long teacher-student lineage during the period of over two hundred years, and there have been expositions and commentaries by many of them, notable among them being *Yukti dīpika* and *Kriyākramakarī* by Śankara and *Gaṇita-yukti-bhāṣā* by Jyeṣṭhadeva which is in Malayalam. An edited English translation of the latter was produced by K.V. Sarma and it has recently been published with explanatory notes by K. Ramasubramanian, M.D.

Srinivas and M.S. Sriram [35]. An edited translation of *Tantrasaṃgraha* has been brought out, more recently, by K. Ramasubramanian and M.S. Sriram [30].

The Kerala works contain mathematics at a considerably advanced level than the earlier works; see [23] and [29] for a mathematical overview and also [10], [11], [12] and [17] for discussions on various aspects of the Kerala works and issues related to them. They include a series expansion for π and the arc-tangent series, and the series for sine and cosine functions that were obtained in Europe by Gregory, Leibnitz and Newton, over two hundred years later. Some numerical values for π that are accurate to 11 decimals are also a highlight of the work. The work of the Kerala mathematicians anticipated the calculus as it developed in Europe later, and in particular it involves manipulations with indefinitely small quantities (in the determination of circumference of the circle etc.) reminiscent of the infinitesimals in calculus; it has also been argued by some authors that the work is indeed calculus already. The overall context raises a question of possible transmission of ideas from Kerala to Europe, through some intermediaries. No definitive evidence has emerged in this respect, but there have been discussions on the issue based on circumstantial evidence; see [17] for instance.

References

- [1] A.K. Bag, *Mathematics in Ancient and Medieval India*, Chaukhambha Orientalia, Varanasi and Delhi, 1979.
- [2] Bhagyashree Bavare, *The Origin of decimal counting: analysis of number names in the Ṛgveda*, *Ganita Bharati* (to appear).
- [3] H.T. Colebrook, *Algebra with Arithmetic and Mensuration, from the Sanscrit of Brahmeḡupta and Bhaṡcara*, John Murray, London, 1817.
- [4] S.G. Dani, *Geometry in Śulvasūtras*, in *Studies in the history of Indian mathematics*, pp. 9-37, *Cult. Hist. Math.* 5, Hindustan Book Agency, New Delhi, 2010.
- [5] B.B. Datta, *The Jaina School of Mathematics*, *Bull. Cal. Math. Soc.*, Vol 21 (1929), No. 2, 115-145.

- [6] B.B. Datta, The Science of the Sulba, a Study in Early Hindu Geometry, University of Calcutta, 1932.
- [7] B.B. Datta and A.N. Singh, History of Hindu Mathematics, A Source Book (2 volumes), Motilal Banarasidas, Lahore, 1935 (Part I) and 1938 (Part II), Asia Publishing House, Bombay, 1962 (reprint), Bharatīya Kala Prakashan, Delhi, 2001 (reprint).
- [8] Joseph Dauben, Book Review of “The Universal History of Numbers and the Universal History of Computing” by Georges Ifrah, Notices of the Amer. Math. Soc. 49 (2002), 32-38.
- [9] Vers une édition critique des *Śulbamīmāṃsā*, commentaries du *Baudhāyana Śulbasūtra*: Contribution à l’histoire des mathématiques sanskrites, Doctoral dissertation, Université Libre de Bruxelles, 2001-2002.
- [10] P.P. Divakaran, The first textbook of calculus: *Yukti-bhāṣā*, Journal of Indian Philosophy 35, 2007, 417-443.
- [11] P.P. Divakaran, Notes on Yuktibhāṣā: recursive methods in Indian mathematics. Studies in the history of Indian mathematics, 287-351, Cult. Hist. Math., 5, Hindustan Book Agency, New Delhi, 2010.
- [12] P.P. Divakaran, Calculus under the coconut palms: the last hurrah of medieval Indian mathematics. Current Science 99 (2010), no. 3, 293-299.
- [13] R.C. Gupta, New Indian values of π from the Mānava Śulba sūtra, Centaurus 31 (1988), no. 2, 114-125.
- [14] R.G. Gupta, Ancient Jain Mathematics, Jain Humanities Press, Tempe, Arizona, USA, 2004.
- [15] Takao Hayashi, The Bakhshālī Manuscript, An ancient Indian mathematical treatise, Groningen Oriental Studies, XI, Egbert Forsten, Groningen, 1995.
- [16] George Ifrah, Universal History of Numbers, Translated from the 1994 French original by David Bellos, E. F. Harding, Sophie Wood and Ian Monk. John Wiley & Sons, Inc., New York, 2000.

- [17] George G. Joseph, *Crest of the Peacock, Non-European roots of mathematics*, Third edition, Princeton University Press, Princeton, NJ, 2011.
- [18] M.C. Joshi (Ed.), *Indian Archaeology 1988-89 - A review*, Archaeological Survey of India, New Delhi, 1993; available online.
- [19] G.R. Kaye, *The Bakhshali Manuscript: A Study In Medieval Mathematics*, 3 Parts, Calcutta 1927; reprinted Aditya Prakashan, 2004.
- [20] Agathe Keller, On Sanskrit commentaries dealing with mathematics (fifth-twelfth century), in *Looking at it from Asia: the Processes that Shaped the Sources of History of Science*, Boston Studies in the Philosophy of Science, 2010, Volume 265, Part 2, 211-244, available at <http://www.springerlink.com/content/u459618708745v1p/>
- [21] Raghunath P. Kulkarni, *Char Shulbasūtra* (in Hindi), Maharshi Sandipani Rashtriya Vedavidya Pratishthana, Ujjain, 2000.
- [22] Padmavathamma, *Gaṇita-sāra-saṃgraha* of Mahāvīra, Original text with Kannada and English translation, Siddhantakīrti Granthamāla of Sri Hom-buja Jain Math, Shimoga District, Karnataka, India, 2000.
- [23] S. Parameswaran, *The Golden Age of Indian Mathematics*, Swadeshi Science Movement, Kerala, 1998.
- [24] Kim Plofker, Mathematics in India, in: *The Mathematics of Egypt, Mesopotamia, China, India and Islam - A Sourcebook*, Ed: Victor J. Katz, pp. 385-514, Princeton University Press, 2007.
- [25] Kim Plofker, *Mathematics in India*; Princeton University Press, Princeton, NJ, 2008.
- [26] Kim Plofker, Links between Sanskrit and Muslim science in Jaina astronomical works, *International Journal of Jaina Studies*, Vol. 6, No. 5 (2010), 1-13.
- [27] Satya Prakash and Ram Swarup Sharma, *Baudhāyana Śulbasūtram*, with Sanskrit Commentary by Dvarakanatha Yajvan and English Translation and Critical Notes by G. Thibaut, The Research Institute of Ancient Scientific Studies, New Delhi, 1968.

- [28] Satya Prakash and Ram Swarup Sharma, *Āpastamba Śulvasūtra*, with commentaries from Kapardswamin, Karvinda and Sundararaja and English translation by Satya Prakash, The Research Institute of Ancient Scientific Studies, New Delhi, 1968.
- [29] K. Ramasubramanian and M.D. Srinivas, Development of Calculus in India, in: *Studies in the history of Indian mathematics*, pp. 201-286, Cult. Hist. Math. 5, Hindustan Book Agency, New Delhi, 2010.
- [30] K. Ramasubramanian and M.S. Sriram, *Tantrasagraha of Nlakaṅṭha Somayājī*, with a foreword by B. V. Subbarayappa, Culture and History of Mathematics, 6. Hindustan Book Agency, New Delhi, 2011.
- [31] M. Rangacharya, *The Ganita-Sara-Sangraha of Mahaviracarya*, Cosmo Publications, 2011.
- [32] R. Salomon, *Indian Epigraphy, A Guide to the Study of Inscriptions in Sanskrit, Prakrit, and the Other Indo-Aryan Languages*, Oxford University Press, 1999.
- [33] Saraswati Amma, *Geometry in Ancient and Medieval India*, Motilal Banarasi-das, Delhi, 1979.
- [34] SaKHYa, *Gaṇitasārakaumudī; the Moonlight of the Essence of Mathematics*, by Ṭhakkura Pherū; edited with Introduction, Translation, and Mathematical Commentary, Manohar Publishers, New Delhi 2009.
- [35] K.V. Sarma, *Gaṇita-yukti-bhāṣā (Rationales in mathematical astronomy) of Jyeṣṭhadeva (Vol. I. Mathematics and Vol. II. Astronomy)*, with explanatory notes by K. Ramasubramanian, M. D. Srinivas and M. S. Sriram, *Sources and Studies in the History of Mathematics and Physical Sciences*. Springer, New York, Hindustan Book Agency, New Delhi, 2008.
- [36] A. Seidenberg, The ritual origin of geometry, *Archive for History of Exact Sciences (Springer, Berlin) Vol. 1, No. 5, January 1975*. (available at: <http://www.springerlink.com/content/r6304ku830258185/>)
- [37] S.N. Sen and A.K. Bag, *The Śulbasūtras*, Indian National Science Academy, New Delhi 1983.

- [38] S.N. Sen, A.K. Bag and S.R. Sarma, A bibliography of Sanskrit works on astronomy and mathematics, National Institute of Sciences of India, 1966.
- [39] Ram Swarup Sharma (Ed.), *Brāhmasphuṭasiddhāntaḥ*, Indian Institute of Astronomical and Sanskrit Research, New Delhi, 1966.
- [40] Kripa Shankar Shukla and K.V. Sarma (Ed.), *Āryabhatīya* of Āryabhata, Critically Edited with Translation, Indian National Science Academy, New Delhi, 1976.
- [41] Kripa Shankar Shukla and K.V. Sarma (Ed.), *Āryabhatīya* of Āryabhata, with the commentary of Bhaskara 1 and Someśvara, Indian National Science Academy, New Delhi, 1976.
- [42] G. Thibaut, The *Śulvasūtras*, The Journal, Asiatic Society of Bengal, Part I, 1875, Printed by C.B. Lewis, Baptist Mission Press, Calcutta, 1875; Edited with an introduction by Debiprasad Chattopadhyaya and published by K P Bagchi and Company, Calcutta, 1984; the original also published as a book, Cosmo Publications, 2012.
- [43] Glen van Brummelen, The mathematics of the heavens and the earth - The early history of trigonometry, Princeton University Press, Princeton, NJ, 2009.
- [44] J.M. van Gelder, *Mānava Śrautasutra*, belonging to the *Maitrāyaṇī Samhita*, International Academy of Indian Culture, New Delhi, 1963.
- [45] Dominik Wujastyk, Indian manuscripts, in *Manuscript Cultures: Mapping the Field*, De Gruyter, Berlin (to appear).

S.G. Dani
 School of Mathematics
 Tata Institute of Fundamental Research
 Homi Bhabha Road
 Mumbai 400005