High Resolution Gamma Ray Spectroscopy

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1 Motivation

Gamma rays emitted from radioactive sources will be measured with a high purity Germanium (HPGe) detector and research-grade electronics. The comparison between a scintillation detector and HPGe detector has been discussed. Finally, identification of different types of interaction of gamma-rays with matter, energy calibration, energy resolution and photopeak efficiency calculation will be performed with HPGe detector, which has been discussed here.

2 Light-Matter Interaction

Of the various ways gamma rays can interact in matter, only three interaction mechanisms have any real significance in gamma-ray spectroscopy: photoelectric absorption, Compton scattering, and pair production. Photoelectric absorption predominates for low-energy gamma rays (up to several hundred keV), pair production predominates for high-energy gamma rays (above 5-10 MeV), and Compton scattering is the most probable process over the range of energies between these extremes [1]. The atomic number of the interaction medium has a strong influence on the relative probabilities of these three interactions. The most striking of these variations involves the cross section for photoelectric absorption. As we shall see from the following discussion, because photoelectric absorption is the preferred mode of interaction, there is a premium on choosing detectors for $\gamma$-ray spectroscopy from materials that incorporate elements with high atomic number.

1. Photoelectric absorption: In this process, a strongly bound atomic electron of the detector material gains kinetic energy by absorbing the incident photon completely. These are called ‘photoelectrons’. For an incident $\gamma$ of frequency $\nu$, the electron with binding energy, $B_e$, gains a kinetic energy,

$$T_e = h\nu - B_e$$  \hspace{1cm} (1)

An output pulse is generated when these photoelectrons are collected at the electrode of a detector. In addition, an X-ray emission takes place when a electron from higher excited level makes a transition to fill the vacancy created by the photoelectron. This X-ray is also absorbed in the detector material. In this way almost complete energy absorption of a photon is possible. The effect of photoelectric absorption is highly dependent on the atomic number of the detector material ($Z$) as well as the $\gamma$ energy, where the absorption cross-section is [2],

$$\sigma \propto \frac{Z^n}{E_{\gamma}^{3.5}}$$  \hspace{1cm} (2)
where, $n= 4$ or $5$ depending on the $Z$ value. Thus, the use of high-$Z$ detector materials increases the photo-peak efficiency.

![Figure 1: Photopeak and Compton continuum are shown in an energy spectrum originating from gamma-matter interaction.](image1)

2. **Compton scattering**: In this process, the $\gamma$-ray transfers a part of its energy to an electron through the scattering process. For a scattering angle, $\theta$, the kinetic energy transferred to the electron is given by,

$$T_e = \frac{E_\gamma^2(1 - \cos \theta)}{m_0c^2 + E_\gamma(1 - \cos \theta)}$$

A detector pulse is produced by the collection of these electrons. The Compton scattered $\gamma$-ray suffers multiple such scatterings unless and until it gets absorbed through photoelectric process or escapes the detector. While the first process contributes to the photopeak, the second process gives rise to a continuous Compton background.

![Figure 2: Relative probabilities of the three types of $\gamma$-ray interaction in Ge & Si as a function of energy.](image2)
3. **Pair production**: The process of pair production takes place in the close proximity of the atomic nucleus for $\gamma$-ray energies exceeding 1.02 MeV. The pair production process creates an electron-positron pair through the spontaneous disappearance of the photon. Both the electron and the positron, lose their kinetic energies through Coulomb scattering, followed by the positron annihilation. A pair of annihilation photons are produced, which either gets absorbed in the detector or escapes. A single escape peak at energy $(E_\gamma - 0.51)$ MeV and a double escape peak at energy $(E_\gamma - 1.02)$ MeV are produced.

3 Gamma Ray Detection

3.1 Scintillation Detectors

The detection of ionizing radiation by the scintillation light produced in certain materials is one of the oldest techniques on record. Each scintillation detector has two components: (i) Scintillator - the material that emits visible light output when exposed on radiation and (ii) light sensors - photomultiplier tubes and photodiodes - which convert the emitted light into an electrical pulse.

The ideal scintillation material should possess the following properties:

1. It should convert the kinetic energy of charged particles into detectable light with a high scintillation efficiency.

2. This conversion should be linear-the light yield should be proportional to deposited energy over as wide a range as possible.

3. The medium should be transparent to the wavelength of its own emission for good light collection.

4. The decay time of the induced luminescence should be short so that fast signal pulses can be generated.

5. The material should be of good optical quality and subject to manufacture in sizes large enough to be of interest as a practical detector.

The scintillation mechanism in inorganic materials depends on the energy states determined by the crystal lattice of the material. Electrons have available only discrete bands of energy in materials classified as insulators, semiconductors and conductors. Valence band represents those electrons that are bound to the lattice sites, whereas the conduction band represents those electrons that have sufficient energy to be free to migrate throughout the crystal. There exists an intermediate band of energies, called the forbidden band, in which electron can never be bound in the pure crystal. Absorption of energy can result in the elevation of an electron from its normal position in the valence band across the gap into the conduction band, leaving a hole in the normally filled valence band. In the pure crystal, the return of the electron to the valence band with the emission of a photon is an inefficient process. Furthermore, typical gap widths are such that the resulting photon would be of too high an energy to lie in the visible range. To enhance the probability of visible photon emission during the de-excitation process small amounts of an impurity are commonly added to inorganic scintillators. Such deliberately added impurities, called activators modifies the band structure from that of the pure crystal.
Because the energy is less than that of the full forbidden gap, this transition can now give rise to a visible photon and therefore serve as the basis of the scintillation process. A charged particle passing through the detection medium will form a large number of electron-hole pairs created by the elevation of electrons from the valence band to the conduction band. The positive hole will quickly drift to the location of an activator site and ionize it, because the ionization energy of the impurity will be less than that of a typical lattice site. Meanwhile, the electron is free to migrate through the crystal and will do so until it encounters such an ionized activator. At this point the electron can drop into the activator site, creating a neutral configuration that can have its own set of excited energy states. These states are illustrated in fig-3 as horizontal lines within the forbidden gap. If the activator state that is formed is an excited configuration with an allowed transition to the ground state, its de-excitation will occur very quickly and with high probability for the emission of a corresponding photon. The decay time of NaI(Tl) is 0.23 \( \mu s \). If the activator is properly chosen, this transition can be in the visible energy range.

A measure of the efficiency of the scintillation process follows from a simple energy calculation. For a wide category of materials, it takes on the average about three times the bandgap energy to create one electron-hole pair. In NaI(Tl), this means about 20 eV of charged particle energy must be lost to create one electron-hole pair. For 1 MeV of particle energy deposited in the scintillator, about \( 5 \times 10^4 \) electron-hole pairs are thus created. Various experimental determinations have shown that the absolute scintillation efficiency of NaI(Tl) is about 12%. Absorption of 1 MeV of energy should therefore yield about \( 1.2 \times 10^5 \) eV in total light energy, or \( 4 \times 10^4 \) photons with an average energy of 3 eV. The yield is thus very close to 1 photon per electron-hole pair originally formed, and the energy transfer to activator sites must be extremely efficient.

A photomultiplier tube then converts this light output to an electrical signal. Two major components inside the tube are a photosensitive layer called the photocathode, coupled to an electron multiplier structure. The photocathode serves to convert as many of the incident light photons as possible into low-energy electrons. If the light consists of a pulse from a scintillation crystal, the photoelectrons produced will also be a pulse of similar time duration. Because only a few hundred photoelectrons may be involved in a typical pulse, their charge is too small at this point to serve as a convenient electrical signal. The electron multiplier section in a PM tube provides an efficient collection geometry for the photoelectrons as well as serving as a near-ideal amplifier to greatly increase their number. After amplification through the multiplier structure, typical scintillation pulse will give rise to \( 10^7 \text{ to } 10^{10} \) electrons, sufficient to serve as the charge signal for the original scintillation event. This charge is conventionally collected at the anode or output stage of the multiplier structure.

A unit of greater significance in scintillation counting is the quantum efficiency (QE) of
The quantum efficiency is simply defined as,

$$\text{QE} = \frac{\text{number of photoelectrons emitted}}{\text{number of incident photons}}$$

(4)

The quantum efficiency would be 100% for an ideal photocathode. Because of the several limitations, practical photocathodes show maximum quantum efficiencies of 20-30%.

### 3.2 Semiconductor Detectors

One of the major limitations of scintillation counters is their relatively poor resolution. The chain of events that must take place in converting the incident radiation energy to light and the subsequent generation of an electrical signal involves many inefficient steps. Therefore, the energy required to produce one information carrier (a photoelectron) is of the order of 100 eV or more, and the number of carriers created in a typical radiation interaction is usually no more than a few thousand. The statistical fluctuations in so small number place an inherent limitation on the energy resolution that can be achieved under the best of circumstances, and nothing can be done about improving the energy resolution beyond this point. The energy resolution for NaI(Tl) scintillators is limited to about 6% when detecting 662 keV gamma rays and is largely determined by the photoelectron statistical fluctuations. The only way to reduce the statistical limit on energy resolution is to increase the number of information carriers per pulse. The use of semiconductor materials as radiation detectors can result in a much larger number of carriers for a given incident radiation event than is possible with any other common detector type.

When a particle deposits energy in a semiconductor detector, equal numbers of conduction electrons and holes are formed within a few picoseconds along the particle track. The detector configurations ensure that an electric field is present throughout the active volume, so that both charge carriers feel electrostatic forces that cause them to drift in opposite directions. The motion of either the electrons or holes constitutes a current that will persist until these carriers are collected at the boundaries of the active volume. So the number of electron-hole pair is proportional to the size of the active volume or depletion depth. Using silicon or germanium of normal semiconductor purity, depletion depths beyond 2 or 3 mm are difficult to achieve despite applying bias voltages that are near the breakdown level. Much greater thicknesses are required for the detectors intended for gamma-ray spectroscopy. The thickness of the depletion region is given by,

$$d = \left(\frac{2\epsilon V}{eN}\right)^{\frac{1}{2}}$$

(5)

where $V$ is the reverse bias voltage and $N$ is the net impurity concentration in the bulk semiconductor material. ($\epsilon$ is the dielectric constant and $e$ is the electronic charge.) At a given applied voltage, greater depletion depths can only be achieved by lowering the value of $N$ through further reductions in the net impurity concentration. Techniques have been developed to achieve the impurity level that is less than 1 part in $10^{12}$ in germanium, but not in silicon. Detectors that are manufactured from this ultrapure germanium are usually called intrinsic germanium or high purity germanium (HPGe) detectors, and they have become available with depletion depths of several centimeters. We will use HPGe detector for this experiment and a brief description of single HPGe detector has been given below.
Figure 4: A portion $^{60}$Co spectrum, illustrating the energy-resolutions and peak-to-Compton ratios for a coaxial HPGe detector compared to a NaI(Tl).

4 Detection with the HPGe Detector

4.1 Detector Structure

Basically, a HPGe detector is a very large semiconductor diode, with a reverse bias voltage applied to its two electrodes to deplete virtually all free charge carriers from the bulk of the detector. Small detectors can be obtained in the planar geometry. The detector is a cylinder of HPGe with electrodes applied to its two circular ends.

Significantly larger detectors benefit from using the coaxial geometry depicted in fig 5. The detector used is a n-type detector composed of a large cylinder of high-purity germanium. A hole is drilled from one end, along the centerline of the cylinder. One electrode is connected to the outer surface of the cylinder while the other electrode is connected to the inside surface of the central hole. The coaxial detector shape is mounted on the end cap of the cryostat, with the symmetry axis of the cylinder aligned coaxially with the centerline of the end cap. The closed end of the detector is located a few millimeters behind the circular surface of the end cap.

The detector and the first amplifying stage of the preamplifier are operated near the boiling temperature of liquid nitrogen (77K) to reduce noise. Consequently, the detector along with the preamplifier are mounted in a vacuum cryostat. The cryostat establishes operation at the desired low temperature via a copper cooling rod dipped in the liquid nitrogen contained in the associated dewar. Operation at the cryogenic temperature dramatically reduces the leakage current in the HPGe detector and also diminishes thermally generated noise in the FET input stage of the preamplifier. The preamplifier feedback capacitor and feedback resistor are also cooled to reduce their noise contribution.
4.2 Pulse Shaping

In dealing with signal pulses from radiation detectors, it is often desirable to change the shape of the pulse in some predetermined fashion. The most common application is in processing a train of pulses produced by a preamplifier. In order to ensure that complete charge collection occurs, preamplifiers are normally adjusted to provide a decay time for the pulse which is quite long (typically 50 µs). If the rate of interaction in the detector is not small, these pulses will tend to overlap one another and give rise to a pulse train that has the appearance shown in Fig. 6a. Because it is the amplitude that carries the basic information (the charge Q deposited in the detector), the "pile-up of pulses on the tails of preceding pulses, which have not fully decayed to zero, can be a serious problem. Because the time spacing between nuclear pulses is random, each pulse can be superimposed on a different residual tail and the resulting amplitude no longer is a good measure of Q from that event.

The ideal solution is to shape the pulses in such a way as to produce a pulse train similar to that shown in Fig. 6b. Here all the long tails have been eliminated, but the information carried by the maximum amplitude of the pulse has been preserved. The pulses have been shaped in the sense that their total length has been reduced drastically but in a way that does not affect the maximum amplitude.

Fig. 7 shows the elements of the basic $CR - RC$ shaping network. An ideal unity gain amplifier separates the two individual networks for impedance isolation so that neither network influences the operation of the other.

In nuclear pulse amplifiers, $CR - RC$ shaping is most often carried out using equal differentiation and integration time constants $\tau$. In that event, output voltage $E_{out}$ at time $t$ for a given step voltage input $E$ is,

$$E_{out} = E \frac{t}{\tau} e^{-t/\tau}$$  \hspace{1cm} (6)
Figure 6: The pulses with long tails shown in part (a) illustrate the apparent variation in amplitude due to pulse pile-up. These effects can greatly be reduced by shaping the pulses as in part (b).

Figure 7: A shaping network consisting of sequential differentiating and integrating stages, denoted as $CR-RC$ network.

Figure 8: The response of $CR-RC$ network to a step voltage input of amplitude $E$ at $t = 0$. Curves are shown for four pairs of differentiator + integrator time constants. Units of the time constants and time scale are identical.
If a single $CR$ differentiation is followed by several stages of $RC$ integration, a pulse shape that approaches a mathematical Gaussian is realized. If the differentiation and $n$ integration time constants are all the same value $\tau$, the particular solution of the corresponding circuit equation is,

$$E_{out} = E \left( \frac{t}{\tau} \right)^n e^{-t/\tau}$$  \hspace{1cm} (7)

In practice, four stages of integration ($n = 4$) are sufficient so that the difference between the resulting pulse shape and a true Gaussian is negligible. The time required for the shaped pulse to reach its maximum amplitude is equal to $n\pi$.

### 4.3 Analog-to-Digital Converters

The critical step in digital pulse processing is the conversion of the analog pulse waveform into digital data. The basic function of an ADC is to produce a digital code (or number) at its output that is proportional to an analog voltage supplied to its input. In the general type of ADC considered, conversions are carried out continuously at a fixed clock frequency. A 100 MHz clock will produce 100 megasamples per second (MSPS), or one sample every 10 ns. In an ideal ADC, each conversion of input voltage to output code is independent, perfectly linear, and occurs instantly. A schematic diagram of basic components used in $\gamma$-ray spectroscopy is shown below.

![Figure 9: Schematic diagram of Gamma-Ray Spectrometer.](image)

![Figure 10: High resolution gamma ray spectroscopy set-up at TIFR, Mumbai.](image)
5 Energy Calibration

1d histogram consists of number of counts with the channel numbers they are digitized in ADC for each HPGe crystal of clover detector. The channel number in ADC is proportional to the deposited energy. So, the relation between the channel number and energy has to be established. This is done by common radioactive source $^{152}$Eu which has known $\gamma$-rays ranging from 121.78 keV to 1408.01 keV (as shown in fig-11).

If $E$ be the energy associated to channel number $x$ then we have calibrated the energy using the relation,

$$E = a + bx + cx^2$$

where, we get the typical coefficients values as the offset coefficient, $a \sim 0.5-1.0$; the slope, $b \sim 0.3-0.4$; and the non-linear coefficient, $c \sim 10^{-8}-10^{-9}$. The plot for energy vs channel number has almost a linear fit. This plot is given below.

![Figure 11: Energy-channel no. calibration fitting and Energy calibrated spectrum of $^{152}$Eu source.](image)

6 Energy Resolution

The energy resolution of HPGe detector is given by,

$$\Delta E_{total} = \sqrt{(\Delta E_{noise})^2 + (\Delta E_{ion})^2 + (\Delta E_{incomplete})^2}$$

where

$$\Delta E_{ion} = 2.35\sqrt{\epsilon FE}$$

$\Delta E_{total}$ is the full width at half maximum amplitude (FWHM) of the gamma-ray peak at energy $E$ in the spectrum. $\Delta E_{noise}$ is the contribution from the noise caused by the detector leakage current and the preamplifier. It is most readily measured as the FWHM of a pulser peak artificially introduced into the spectrum by injecting a pulser signal into the input of the preamplifier. The noise contribution is independent of the gamma-ray energy. But, it does depend on the shaping time constant of the spectroscopy amplifier. If the shaping time constant is too small or too large, the noise contribution will be higher than the optimum. Check the detector data sheet for the optimum shaping time constant to minimize the noise. The optimum will likely lie in the range of 3 to 6 microseconds.
\( \Delta E_{\text{ion}} \) describes the variation in the number of electron-hole pairs generated as a result of ionization statistics. It depends on the average energy required to create an electron-hole pair (i.e., \( \epsilon = 2.95 \text{ eV} \)), the energy of the gamma-ray, \( E \), and the Fano factor, \( F \). Note that the same units of energy must be used throughout eq-10. The Fano factor accounts for the fact that the ionization process lies somewhere between completely independent random ionization events at one extreme (\( F = 1 \)), and an absolutely deterministic conversion of energy into electron-hole pairs at the other extreme (\( F = 0 \)). For HPGe a Fano factor, \( F \sim 0.1 \), indicates the process is closer to the latter than the former condition.

\( \Delta E_{\text{incomplete}} \) accounts for the variation in the ability to collect all of the electron-hole pairs that are created by the ionization process. Primarily, this applies to electron-hole pairs that recombine before they can be collected, or charge carriers that fall into traps while drifting to their respective electrode. For large coaxial HPGe detectors the charge collection time can vary from 50 to 700 ns, depending on the position at which the charge was created. If the amplifier shaping time constant is not large compared to these collection times, the pulse height will show additional fluctuations caused by random variations in the charge collection times. If the incomplete charge collection term is ignored in eq-10, spectral resolution measurements may lead to an inflated value for the implied Fano factor.

Comparison between the energy spectra for \(^{60}\text{Co}\) tested by the best scintillator \( \text{LaBr}_3 \) (Ce) and HPGe detector also shows that the energy resolution in HPGe is much better than \( \text{LaBr}_3 \).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure12.png}
\caption{\(^{60}\text{Co}\) energy spectrum, illustrating the energy-resolutions and peak-to-Compton ratios for HPGe detector compared to \( \text{LaBr}_3 \) (Ce).}
\end{figure}

7  Photopeak Efficiency Calibration

Absolute photopeak efficiency (\( \epsilon \)) is defined as the ratio of counts in a photopeak to the number of gamma emitted by the source. As efficiency varies with the \( \gamma \)-energy (\( E_\gamma \)), therefore, efficiency calibration is done using same \(^{152}\text{Eu}\) source.

If \( Y \) is area under the photopeak and \( Y_M \) is the standard area of that peak available in RADWARE [3], then relative photopeak efficiency, \( \epsilon \) is calculated by the formula,

\[ \epsilon_\gamma = \frac{Y}{Y_M} \]  \hspace{1cm} (11)
and the formula used to fit the efficiency $\epsilon$ with photopeak energy $E$ is given by,

$$\epsilon(E) = a + \frac{b}{E} + \frac{c}{E^2}$$  \hspace{1cm} (12)

A more complicated formula is found in RADWARE [3] as,

$$\ln(\epsilon) = [(A + Bx + Cx^2)^{-G} + (D + Ey + Fy^2)^{-G}]^{-1/G}$$  \hspace{1cm} (13)

where $x = \ln(100/E_\gamma)$ and $y = \ln(1000/E_\gamma)$. A typical energy-efficiency curve plotted by eq-12 for $^{152}\text{Eu}$ source is shown in fig-13.

![Figure 13: Typical Efficiency calibration plot for $^{152}\text{Eu}$ source.](image)

8 Decay Schemes for Radioactive Sources [4]

![Figure 14: Decay schemes of radioactive $^{60}\text{Co}$ and $^{137}\text{Cs}$ sources, respectively.](image)
9 Lab Exercises

1. Energy calibration of ADC of each HPGe Clover detector using $^{133}$Ba and $^{152}$Eu sources.

2. Identification of $\gamma$-rays from unknown sources.

3. Estimation of photopeak detection efficiency using $^{133}$Ba and $^{152}$Eu sources.

4. Plot FWHM vs $E_\gamma$ and variation of the photo peak energy resolution with shaping time of the amplifier.

5. Identification of the background radiation in the room.

6. Comparison of the $\gamma$-ray spectrum between HPGe and LaBr$_3$(Ce) detectors.

References


