Magnetic field induced emergent inhomogeneity in a superconducting film with weak and homogeneous disorder

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When a magnetic field is applied on a conventional type-II superconductor, the superconducting state gets destroyed at the upper critical field, $H_c^*$, where the normal vortex cores overlap with each other. Here, we show that in the presence of weak and homogeneous disorder the destruction of superconductivity with the magnetic field follows a different route. Starting with a weakly disordered NbN thin film ($T_c \sim 9$ K), we show that under the application of a magnetic field the superconducting state becomes increasingly granular, where regions filled with chains of vortices separate the superconducting islands. Consequently, phase fluctuations between these islands give rise to a field induced pseudogap state, which has a gap in the electronic density of states, but where the global zero resistance state is destroyed.

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I. INTRODUCTION

Over the past decade, the notion of emergent granularity has evolved from a theoretical proposition [1–3] to an alternative paradigm to understand the superconducting state in the presence of strong homogeneous disorder [4–10]. Clean $s$-wave superconductors which are well described by Bardeen-Cooper-Schrieffer (BCS) theory [11] are characterized by an energy gap ($\Delta$) in the electronic density of states (DOS) centered around the Fermi level, and two sharp coherence peaks at the gap edge. Theoretically, $\Delta$ is the Cooper pairing energy scale which determines the superconducting transition temperature, $T_c$, whereas the coherence peaks signify the establishment of long-range phase coherence. Since the late 1950s [12,13], it was known that $\Delta$ remains finite even at strong (nonmagnetic) disorder, leading to the belief that the superconducting transition would also remain robust against disorder. However, this conclusion is invalidated by the emergence of granularity. It is now understood that in the presence of strong disorder, the superconducting state can segregate into superconducting and insulating regions, where zero resistance is achieved through Josephson tunneling between superconducting islands. Consequently, the superconducting state can get destroyed through phase fluctuations between the islands, even when the pairing amplitude remains finite [1,14]. Experimentally, this manifests as a pseudogap [15,5,6,16], which persists well above $T_c$, where the zero resistance state is destroyed. Experimental and theoretical evidence also suggests that at a critical disorder Cooper pairs can eventually get localized, giving rise to an insulator made out of Cooper pairs [17,18].

Another aspect of strongly disordered superconductors, namely, the magnetic field induced superconductor-insulator transition (SIT) [19,20], has also attracted considerable attention. Under the application of a magnetic field, the ground state of several strongly disordered superconductors transform into an insulator, with sheet resistance exceeding $10^9$ $\Omega$. While no consensus has so far emerged on the origin of this insulator state, it is now widely accepted that it is related in some way to the superconducting correlations [21–23]. Several theoretical scenarios, such as Coulomb blockade in the emergent granular superconducting state [24] and boson localization [25], as well as theories invoking charge vortex duality [26,27], have been proposed to explain this phenomenon. From an experimental standpoint it is therefore important to obtain microscopic information on the evolution of the superconducting state with magnetic field in order to discriminate between various possibilities.

Here, using low-temperature scanning tunneling spectroscopy (STS) we investigate the magnetic field evolution of the superconducting state in a weakly disordered NbN thin film [28]. The sample under investigation is an NbN film with $T_c \sim 9$ K, corresponding to $k_F l \sim 4$ (where $k_F$ is the Fermi wave vector and $l$ is the electronic mean free path) [5]. For this $k_F l$, the coherence length [29] $\xi \sim 5–10$ nm, and the magnetic penetration depth [28] $\lambda \sim 800$ nm. It has been shown earlier [28] that by controlling deposition parameters the disorder in NbN films can be tuned over a large range, from $k_F l \sim 10 \sim k_F l \sim 0.42$. As the disorder is increased the superconductor progressively passes through three regimes: Regime I ($10 \gtrsim k_F l \gtrsim 3$) where the superconducting energy gap in zero field vanishes at the same temperature where resistance appears; Regime II ($3 \gtrsim k_F l \gtrsim 1$), where a pronounced pseudogap state appears above $T_c$; and Regime III ($k_F l \lesssim 1$), where superconductivity is completely suppressed down to 300 mK. In the present context, “weak disorder” refers to Regime I where the zero-field state follows the BCS paradigm. However, it is important to note that in the BCS sense, NbN films in general (in all three regimes) are in the strong disorder limit [30], where $\xi_{BCS} = \frac{\hbar v_F}{2\pi k_F} \gg l$ ($v_F$ is the Fermi velocity). The normal state exhibits a weak negative temperature coefficient of resistance with $\frac{R(300 \text{ K})}{R(15 \text{ K})} \sim 0.78$, consistent with earlier reports at this level of disorder [31]. The central result of this paper is that when a magnetic field ($H$) is applied on the sample, the superconducting state becomes inhomogeneous, in a manner similar to what disorder alone would have done.

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at a much larger strength. This experimental observation is backed by numerical simulations which show that the flux tubes enter a disordered superconductor at locations where disorder partially suppresses the superconducting correlations. This disorder creates a network of weak links where vortices enter, and additionally suppress the superconducting order parameter. The resulting superconducting state contains regions, a few tens of nanometers in size, where the superconducting order parameter is finite, separated by chains of vortices where the superconducting order is suppressed. Consequently, the system exhibits a field induced pseudogap that progressively widens as the magnetic field is increased.

II. SAMPLE DETAILS AND EXPERIMENTAL METHODS

Sample. The epitaxial NbN thin film with thickness ~25 nm was deposited on single-crystalline MgO (100) substrate using reactive pulsed laser deposition using a KrF excimer laser (248 nm). A pure Nb target was ablated in an ambient N2 atmosphere of 40 mTorr, while the substrate was kept at 600 °C. For the ablation process, laser pulses with energy density 200 mJ/mm^2 were used with a repetition rate of 10 Hz. To maintain a pristine surface while transporting the sample for STS measurements, the sample was transferred in situ in an ultrahigh-vacuum suitcase with base pressure ~10^{-10} Torr and transferred to the low-temperature scanning tunneling microscope (STM) without exposure to air. Transport (and magnetic) measurements were performed on the sample after all STS measurements were completed. The resistance measurements as a function of temperature and magnetic field were performed using conventional four-probe technique, using a current of 0.5 mA.

Scanning tunneling spectroscopy measurements. All STS measurements were performed in a home-built STM [32] operating down to 350 mK and fitted with a superconducting solenoid with maximum field of 90 kOe. The STM tip was made out of a mechanically cut Pt-Ir tip, which was sharpened in situ by field emission on an Ag single crystal. The tunneling conductance \( G(V) = \frac{dI}{dV} \) was measured by adding a 150-μV, 2-kHz ac voltage to the dc bias voltage \( V \) and recording the ac response in the tunneling current \( I \) using standard lock-in technique. For the conductance maps at fixed bias voltage, the ac response in the tunneling current was measured while the tip was rastered over the sample surface. The full area spectroscopy was performed by stabilizing the tip at every point, then momentarily switching off the feedback loop and sweeping \( V \) from +5 mV to −5 mV and recording the ac response in the tunneling current as a function of bias voltage.

III. RESULTS

A. Zero-field superconducting state

The tunneling conductance between a normal tip and a superconductor, measured using an STM, provides the most direct access to the local density of states in a superconductor. We characterize the zero-field state by measuring the tunneling conductance spectra \( G(V) \) over a 200-nm × 200-nm area at 450 mK. The tunneling conductance reveals a uniform energy gap and the presence of a coherence peak at the gap edge over the entire area. The representative spectra along a 200-nm line are shown in Fig. 1(a). To look at the spatial variation of the zero bias conductance (ZBC) and the coherence peak height we normalize the tunneling spectra with \( \Delta(T) \) obtained from BCS theory. The blue line shows temperature variation of resistance measured on the same sample. The resistance appears exactly at the same temperature, \( T_c \), where the BCS gap vanishes.

FIG. 1. Superconducting state in zero field. (a) Representative normalized tunneling conductance spectra \([G_N(V) vs V]\) in zero field at 450 mK along a 200-nm line. The spectra show a uniform energy gap and a finite coherence peak at the gap edge over the entire line. (b) Coherence peak height \((G_{Np})\) map over a 200-nm × 200-nm area, forming an inhomogeneous structure. (c) Temperature variation of the average superconducting energy gap \( \Delta \) (green squares), and broadening parameter \( \Gamma \) (red circles) obtained by fitting the average \( G_N(V) vs V \) spectra. The fits to the tunneling spectra are shown in the inset. The light green line is the expected variation of \( \Delta(T) \) obtained from BCS theory. The blue line shows temperature variation of resistance measured on the same sample. The resistance appears exactly at the same temperature, \( T_c \), where the BCS gap vanishes.
FIG. 2. Superconducting state in magnetic field. (a)–(c) Conductance maps at 40, 60, and 75 kOe respectively, taken at 450 mK at fixed bias 2.2 mV over an area of 200 \text{nm} \times 200 \text{nm}. The red dots show the position of the vortices obtained from the local minima in the conductance. (d)–(f) Normalized tunneling spectra along the green lines shown in the conductance maps (a)–(c), respectively, which pass through the center of the vortices. The vertical black dashed line denotes the center of the vortex. (g)–(i) Representative spectra for three different fields, respectively, at the center of the vortex core (black) and away from the core (violet) corresponding to the black and violet dashed lines shown in panels (d)-(f). In contrast to a conventional superconductor, we observe a soft gap at the core of the vortices.

We now turn our focus to the temperature variation of \( \Delta \). To obtain \( \Delta \), we fit the \( G_N(V) \) vs \( V \) spectra averaged over the entire area [Fig. 1(e)] with the tunneling equation, 
\[
G(V) \propto \int_{-\infty}^{\infty} N_s(E) \left[ \frac{\partial f(E - eV)}{\partial E} \right] dE.
\]
Here \( e \) is the electronic charge, \( f(E) \) is the Fermi Dirac distribution function, and \( N_s(E) = \sqrt{E + i/\Gamma_1} / \sqrt{E + i/\Delta} \) is the BCS quasiparticle DOS, where a phenomenological broadening parameter \( \Gamma_1 \) is incorporated to take into account nonthermal sources of broadening in the DOS [34]. Comparing with the temperature variation of resistance, we observe that \( \Delta \) follows the usual BCS temperature dependence and vanishes at \( T_c \). \( \Gamma_1 \) on the other hand is nearly temperature independent and varies between 0.23 and 0.33. The dominant role of \( \Gamma_1 \) is to broaden the coherence peak and suppress its height. The relatively large value of \( \Gamma_1 \) obtained by fitting the average spectra reflects the presence of regions where the coherence peak is suppressed.

**B. Emergence of granular superconducting state in magnetic field**

When a magnetic field is applied, the field enters a type-II superconductor in the form of vortices comprising a circulating supercurrent and enclosing a magnetic flux quantum, \( \Phi_0 = \hbar/2e \). At the center of each vortex is the vortex core, where the superconducting order is destroyed and the circulating supercurrent is zero. In an STS experiment, these cores can be identified by recording the conductance map with bias voltage close to the coherence peak, where the vortices appear as low-conductance points owing to the suppression of the coherence peak [35]. To identify the vortex cores in our sample, we record the conductance maps at 450 mK in different magnetic fields (\( H \perp \text{film surface} \)), at a fixed bias voltage of 2.2 mV [Figs. 2(a)–2(c)]. The vortices, corresponding to the local minima in the conductance, are shown as red dots. In this context, we would like to note that with increasing
field, we also observe a decrease in the maximum value of conductance even far from the vortex core. This is a direct result of the orbital supercurrent, which in an extreme type-II superconductor (\(\lambda \gg \xi\)), extends well beyond the core of the vortex up to a length of the order of \(\lambda\), and causes partial suppression of the coherence peaks. Therefore the upper limit of the color scale in Figs. 2(a)–2(c) is adjusted to the maximum value of the conductance within the area and the depth of the color is kept the same in all three panels, such that each image is effectively normalized to the maximum conductance value within the area. The area was kept the same for different fields and small drifts (\(<10\,\text{nm}\)) were corrected by looking at topographic features of the surface. To avoid large drifts during large temperature sweeps, the sample was first cooled in zero field and the data were taken by gradually ramping up the field and stabilizing at specific values. In principle, this zero-field-cooled (ZFC) protocol could suffer from the drawback that in a disordered superconductor the flux density at the center of the sample might be lower than the applied field owing to strong pinning. To verify whether the magnetic field is uniformly entering at the fields where we performed the STS measurements, careful magnetization measurements were performed on the field-cooled (FC) and ZFC state as a function of temperature. We observed that the magnetization of the two states becomes indistinguishable above 3 kOe (see Appendix A), confirming that the flux completely penetrates the sample above this field. At 40 kOe we observe that chains of vortices form a laminar structure, separating regions where \(G(V) = 2.2\,\text{mV}\) is high. As the field is increased, further entry of vortices progressively widens the regions with suppressed coherence peak, and the laminar structure becomes denser, thereby shrinking the puddles where the coherence peak is high. We would like to note that at 60 and 75 kOe, when counting the number of vortices we observe that multiplying the number of vortices with \(\Phi_0\) does not account for the entire flux expected to pass through the area for the applied magnetic field. This discrepancy arises because of our inability to account for two very closely placed vortices which appear as a single patch where the coherence peak is suppressed. This is consistent with numerical simulations that we present later.

We now investigate the nature of the vortex core, by recording the tunneling spectra along a line passing through the center of a vortex [Figs. 2(d)–2(f)]. Due to the uncertainty in the number of vortices in every patch at 60 and 75 kOe a direct comparison of these line scans is difficult. Nevertheless, all three line scans display a surprising common feature. In a conventional superconductor, the core of an Abrikosov vortex behaves like a normal metal, where the tunneling spectrum is either flat [\(G_N(V) \sim 1\)] or displays a small peak at zero bias owing to the formation of Caroli–De Gennes–Matricon [36] (CDM) bound states in very clean samples (see Appendix B). In contrast here, we observe that a soft gap continues to survive even at the center of the vortex core [Figs. 2(g)–2(i)] although the coherence peak gets suppressed [37]. The suppression of the coherence peak suggests that the superconducting order parameter is suppressed in the core of the vortex even though the pairing amplitude remains finite.

Since the proliferation of vortices locally suppresses the superconducting order, the inhomogeneous distribution of vortices produces boundaries of suppressed superconductivity which separate the superconducting patches. To visualize this superconducting state more clearly, we measured the tunneling conductance spectra over the same area on a \(32 \times 32\) grid in different magnetic fields. Figures 3(a)–3(d) show the \(G_N(0)\) maps at different fields up to 75 kOe corresponding to the same area as in Figs. 2(a)–2(c). We observe that with increase in field the superconducting state develops large inhomogeneity, forming regions where \(G_N(0)\) is large and regions where \(G_N(0)\) is small. This is also reflected in the distribution of \(G_N(0)\), which with increase in field develops large tails [Figs. 3(e)–3(h)]. Figures 3(i)–3(k) show the coherence peak height maps corresponding to the same fields. We observe an inverse correlation of the \(G_Np\) maps with the \(G_N(0)\) maps, implying that in regions where \(G_N(0)\) is large, the coherence peak is suppressed. The anticorrelation is also apparent from the two-dimensional histogram of \(G_N(0)\) and \(G_Np\) which shows a negative slope over a large scatter, which suggests that the anticorrelation is not perfect. We quantify the anticorrelation using the cross correlator,

\[
I = \frac{1}{n} \sum_{i,j} \frac{(G^i,j_N(0) - \langle G_N(0) \rangle)(G^i,j_Np - \langle G_Np \rangle)}{\sigma_0 \sigma_p},
\]

where \(\sigma_0\) and \(\sigma_p\) are the standard deviations in the values of \(G_N(0)\) and \(G_Np\), respectively; \(i, j\) refer to the pixel index of the image; and \(n\) is the total number of pixels. We obtain \(I \sim -0.15\) to \(-0.2\) where \(I = -1\) implies perfect anticorrelation. This is qualitatively similar to earlier observation in strongly disordered NbN samples in zero field [7]. The weak anticorrelation suggests that \(G_N(0)\) is probably not governed by the local superconducting order parameter alone. As expected, the vortices (shown in red rods) are preferentially located in the regions where \(G_N(0)\) is high.

C. Field induced pseudogap state

We now investigate the temperature evolution of the superconducting state in a magnetic field. Figures 4(a)–4(e) show the temperature variation of the average \(G_N(V)\)-V spectra over a \(200\,\text{nm} \times 200\,\text{nm}\) area at different magnetic fields along with the temperature variation of resistance. As the magnetic field is increased we observe that a soft gap in the tunneling spectrum continues to persist up to a temperature \(T^*\), well above \(T_c(H)\). (For consistency, we define \(T^*\) as the temperature where \(G_N(0)\) is 95% of the normal state value.) Here, \(T_c(H)\) is defined as the temperature where the resistance is 0.05% of its normal state value (for more details, see Appendix C). This is analogous to the pseudogap state observed earlier in zero field in strongly disordered superconductors [4,5,7,15]. Plotting \(T_c(H)\) and \(T^*\) in the \(H-T\) parameter space [Fig. 4(f)], we observe that the pseudogap state becomes progressively wider as the magnetic field is increased. To rule out the possibility that the observed pseudogap is caused from a local distribution in temperature at which \(\Delta \to 0\), we have also separately tracked the temperature dependence of the tunneling spectra at locations where the coherence peak at low temperature is finite and locations where the coherence peak is suppressed. Figures 4(h)–4(i) show the temperature evolution corresponding to two such locations [Fig. 4(g)] at 40 kOe. We observe that at both
FIG. 3. Magnetic field induced granularity. (a)–(d) ZBC $G_N(0)$ maps for fields 0, 40, 60, and 75 kOe, respectively, over the same 200-nm $\times$ 200-nm area at 450 mK, obtained from area spectroscopy over a 32 $\times$ 32 pixels grid. The red dots show the positions of the vortices. (e)–(h) Distribution of $G_N(0)$ for 0, 40, 60, and 75 kOe, respectively. With increasing field the distributions develop large tails, signifying emerging inhomogeneity with field. (i)–(k) Coherence peak height ($G_{Np}$) maps for 40, 60, and 75 kOe. The upper limit of the color scale is set to the maximum value of $G_{Np}$ at that field. (l)–(n) Cross-correlation histograms between $G_N(0)$ and $G_{Np}$ for corresponding fields, showing inverse correlation between the two quantities; in all these three histograms the bin size is adjusted to segment $G_N(0)$ and $G_{Np}$ into 15 and 18 bins, respectively, over their plotted range.

locations $G_N(0) \to 1$, at the same temperature, confirming that the pairing amplitude uniformly disappears at the same temperature [Fig. 4(j)].

IV. COMPARISON WITH NUMERICAL SIMULATIONS

We next carry out numerical simulation in order to develop further insight into our experimental findings. We describe our system through an attractive Hubbard Hamiltonian, which in the presence of disorder and applied magnetic field has the form

$$H = -t \sum_{\langle i,j \rangle,\sigma} e^{i \phi_{ij}} c_{i\sigma}^+ c_{j\sigma} - |U| \sum_{i} \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} + \sum_{i,\sigma} (V_i - \mu) \hat{n}_{i\sigma},$$

(1)

where $c_{i\sigma}$ ($c_{i\sigma}^+$) annihilates (creates) an electron with spin $\sigma$ at site $i$ of a two-dimensional square lattice, $\hat{n}_{i\sigma} = c_{i\sigma}^+ c_{i\sigma}$ is the occupation number of site $i$ with spin $\sigma$, and the phases $\phi_{ij} = \frac{\pi}{\Phi_0} \int_{B} A \cdot d\ell$ are the Peierls factor of an applied orbital magnetic field. We use the Landau gauge $\vec{A} = B x \hat{y}$ for all our calculations. The attraction $U$ induces $s$-wave
FIG. 4. Field induced pseudogapped state. (a)–(e) Temperature variation of the average $G_N(V)$-V spectra at 0, 20, 40, 60, and 75 kOe, respectively, along with the temperature variation of resistance. The vertical dashed lines correspond to $T_c$, where resistance appears and $T^*$, where the pseudogap in the density of states disappears. At $H = 0$ these two happen at the same temperature. The range of the temperature axes in all plots has been kept the same for visual comparison. (f) $T_c$ and $T^*$ are plotted on the $H$-$T$ space, which shows that the pseudogap state widens as the field is increased. (g) Conductance map at fixed bias; $V = 2.2$ mV at 40 kOe at 450 mK. The blue and green boxes show two representative areas where the conductance is high and low, respectively. (h),(i) Average state widens as the field is increased. (g) Conductance map at fixed bias; $V = 2.2$ mV at 40 kOe at 450 mK. The blue and green boxes show two representative areas where the conductance is high and low, respectively. (j) Temperature dependence of $G_N(0)$ for the average spectra inside the blue and green box, respectively; we observe that in both cases $G_N(0)$ goes to 1 at the same temperature.

superconductivity in the system. The disorder at site $i$ is given by $V_i$, which is chosen as an independent random variable from a uniform distribution between $-V$ and $V$ which quantifies the disorder strength $V$. The chemical potential $\mu$ fixes the average density $\langle \rho \rangle = (1/N) \sum_i \hat{n}_i$, where $\hat{n}_i$ is the occupancy of the $i$-th site and $N$ is the total number of sites) which we fix at $\rho = 0.875$ for all our calculations. We carry out a fully self-consistent mean field analysis of (1) using the Bogoliubov–de Gennes (BdG) technique following Refs. [38–40] on a $36 \times 36$ two-dimensional grid. Considering the lattice spacing of NbN, which is $\sim 4.4$ Å, the size of our simulation would translate to an area of $16 \text{nm} \times 16 \text{nm}$. The small size of the simulation is necessitated due to available computational resources and results in two caveats. First, we need to use a large value of $|U|$, beyond the weak-coupling BCS value, to keep the coherence length well within the system size. We use $|U| = 1.2\xi$, which is the clean limit gives a coherence length, $\xi \sim 10-12$ lattice spacing [38]. This is further reduced in the presence of disorder yielding an operational coherence length, $\xi \sim 5-6$ lattice spacing. This is also consistent with the dirty limit relation, $\xi \sim (\xi_0)^{1.5}$. Secondly, due to the small simulation area the effective magnetic field for a given number of vortices (n) is much larger than the experimental value. While this drawback prevents quantitative comparison with the experimental data, it has been shown through several studies that the finite size simulations capture the broad qualitative features of disordered superconductors [1,2,24,33,38]. Since these simulations are restricted to $T = 0$, we compare our simulations with experimental data taken at the lowest temperature, 450 mK. (For further details see, Appendix D.)

We first investigate the zero-field state obtained from the simulations. Figure 5(a) shows the single-particle DOS, $D(E)$, normalized to its value at $E = -0.2t$, averaged over the entire lattice for $V = 0.5t$. The average $D(E)$ shows a fully formed gap and sharp coherence peaks consistent with the zero-field tunneling spectra at 450 mK. Figure 5(b) shows the spatial variation of $D(E)$ at the coherence peak, $D_p$. We observe that $D_p$ shows large spatial variation forming an inhomogeneous structure similar to that in Fig. 1(b). To confirm that this disorder strength $V$ is indeed appropriate for our experiments, we compare the width of the normalized distribution of $D_p$ at 450 mK in zero field, defined as $\tilde{G}_{Np} = \frac{G_{Np} - G_{Np}^{\text{min}}}{G_{Np}^{\text{max}} - G_{Np}^{\text{min}}}$ with the corresponding normalized distribution of $D_p$, namely, $\tilde{D}_p = \frac{D_p - D_p^{\text{min}}}{D_p^{\text{max}} - D_p^{\text{min}}}$ [where $G_{Np}^{\text{min}} (D_p^{\text{min}})$ and $G_{Np}^{\text{max}} (D_p^{\text{max}})$ are
the minimum and maximum of $G_{NP}$ ($D_p$). The distributions [Fig. 5(c)] have similar width as measured from standard deviations from the mean value showing that we are working at comparable disorder strength.

We now track the evolution of the superconducting state with magnetic field. Since the vortex simulations are carried out by repeating the simulation box periodically, $n$ can only be even [39]. Figures 6(a)–6(c) show the spatial variation of the phase of the superconducting order parameter $\phi$ for $n = 2, 4,$ and $6$; the color scale corresponds to the local pairing amplitude defined as $|\Psi| = |U| |c_1 c_1^\dagger|$. The positions of vortex cores can be identified from the locations where $\phi$ twists around a point and $|\Psi| \approx 0$. The spatial variation of $D(0)$ and $D_p$ corresponding to these flux fillings shown in Figs. 6(d)–6(i) qualitatively captures all the broad features observed in our experiment. The presence of the vortex results in a local increase in $D(0)$. From the map of $D_p$, for $n = 2$ we clearly see that the spatial variation of $D_p$ is anticorrelated with $D(0)$; i.e., $D_p$ is small at locations where $D(0)$ is large. This anticorrelation is weaker for $n = 4$, and not discernible for $n = 6$. This is in contrast with the experimental data where the anticorrelation is nearly independent of magnetic field. This disagreement with experiment is possibly due to the very high effective magnetic field for $n = 4$ and $n = 6$ due to the small size of our simulation. Furthermore, we observe from the $D(0)$ maps that the regions with large $D(0)$ around two closely located vortices coalesce to form one continuous larger patch. (The length scale of these patches is of the order of the coherence length $\xi$ in our simulation.) Therefore, it is likely that at high fields we are unable to resolve all the individual vortices from the conductance images, which accounts for the apparent nonconservation of magnetic flux in our sample. We also observe that the distribution of $D(0)$ progressively increases with increasing $n$ [Figs. 6(j)–6(l)] and forms long tails consistent with experiments.

Finally, we dwell on the issue of the soft gap observed inside the vortex core in our experiments. In Fig. 6(k) (inset) we compare the average $D(E)$ close to the center of the vortices and at regions far from it, for $n = 4$. Close to the center of the vortices the coherence peak is completely suppressed but a soft gap continues to survive. For a consistency check we have performed the same calculations without any disorder ($V = 0$) (see Appendix D). In that case, we realize an Abrikosov lattice commensurate with the lattice geometry, and $D(E)$ shows a large zero energy peak inside the vortex core consistent with the CDM bound state [36]. To understand physically the origin of this behavior we note the circulating supercurrent density around a vortex, $J \propto (1/r)$, where $r$ is the radial distance from the center of the vortex. In a clean superconductor, the normal vortex core appears below a limiting $r$, where the increase in kinetic energy of the Cooper pair exceeds the pairing energy, $2\Delta$, and destroys the superconducting pairing. In the presence of disorder a completely different scenario can emerge. Here, disorder scattering reduces the superfluid stiffness, $J_s$, making the superconductor susceptible to phase fluctuations [28]. The survival of the soft gap in the vortex core, in our opinion, is strongly tied to the phase fluctuations of the order parameter due to the inhomogeneous background that depletes the superfluid stiffness in the core regions, but keeps the pairing amplitude finite.

V. SUMMARY AND OUTLOOK

The emerging physical picture from the experiments and the simulations is as follows: The random disorder potential makes the superconducting order parameter spatially inhomogeneous even in the absence of magnetic field. As a result, the flux tubes from the applied field thread the system through locations where the local amplitude of the order parameter, $|\Psi|$, is low. Such spatial organization lessens the energy cost by accumulating the phase-twist in regions of low $|\Psi|$. This naturally makes the field induced vortex lattice aperiodic, and flux tubes wipe out remnants of pairing amplitude in a region of size $\sim \xi$ around vortex centers. Such local annihilation of superconducting correlations with magnetic field introduces granularity in the superconducting state even with low disorder strengths, in a manner similar to what disorder alone would have done for much larger strengths. Consequently, the pseudogap is observed in the region of $H-T$ parameter space where Cooper pairs continue to survive even when the zero

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FIG. 6. Simulation in the presence of vortices. (a)–(c) The spatial variation of $\phi$ shown as arrows for $n = 2, 4,$ and 6 vortices on the $36 \times 36$ lattice. The colors of the arrows stand for the strength of superconducting order parameter $|\Psi|$ on a scale of 0 to 1 on the lattice. The positions of the vortices can be identified from the locations where $\phi$ twists around a point and $|\Psi|$ has a value close to zero. (d)–(f) The spatial variation of $D(0)$ corresponding to $n = 2, 4,$ and 6 vortices on the $36 \times 36$ lattice. (g)–(i) The spatial variation of $D_p$ corresponding to $n = 2, 4,$ and 6 vortices on the $36 \times 36$ lattice. (j)–(l) Distribution of $D(0)$ for $n = 2, 4,$ and 6 vortices, respectively. With increasing field the distributions develop large tails, signifying emerging inhomogeneity with field. The inset of (k) shows the $D(E)$ as a function of $E/t$ for $n = 4$ averaged over regions close to the center of the vortices (black) and for regions far from the vortices (blue).
resistance state is destroyed due to phase fluctuation between superconducting puddles. This is consistent with earlier planar tunneling measurements on Pb-Bi films [41].

Our results provide valuable clues to understanding the magnetic field induced superconductor-insulator transition in much more strongly disordered samples, which are very close to, but on the superconducting side of, disorder driven SIT. There, even the zero-field state consists of regions where $|\Psi|$ is completely suppressed such that the superconducting state is composed of superconducting puddles that are Josephson coupled through insulating regions. When a magnetic field is applied, the superconducting puddles will further fragment through vortex proliferation, until they reach a critical size where Coulomb blockade makes it energetically unfavorable for the current to pass through the superconducting islands [24]. At this point we would expect to see a transition from a superconductor to an insulatorlike behavior in transport measurements. Therefore, we propose that the disorder and magnetic field driven SITs are both manifestations of the same microscopic phenomenon: the granularity that emerges naturally in the superconducting state. Microscopic validation of this scenario could be obtained through STS measurements on more strongly disordered superconductors at very low temperatures.

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R.G. and I.R. performed the measurements. I.R. and R.G. analyzed the data. H.S. prepared the sample. A.B. carried out the simulations under the supervision of A.G. P.R. conceived the problem and supervised the experiments. P.R. and A.G. wrote the paper. All authors discussed the results and commented on the manuscript. R.G. and I.R. contributed equally as joint first authors of this paper.

APPENDIX A: FLUX PENETRATION IN THE SUPERCONDUCTOR IN THE ZERO-FIELD-COOLED STATE

In a strongly pinned superconductor, when a magnetic field is applied after the superconductor is cooled to low temperatures in zero magnetic field [i.e., the zero-field-cooled (ZFC) state], the entry of magnetic flux is hindered by the pinning potential. Consequently, the flux density gradually decays from the edge towards the center of the superconductor, and the resulting flux density gradient is determined by the local critical current density [42]. In contrast, when the sample is cooled from above $T_c$ in the presence of a magnetic field [i.e., the field-cooled (FC) state] the presence of strong pinning traps the magnetic flux threading the sample in the normal state in the form of vortices, producing a nearly uniform flux density profile, and a magnetization ($M$) that is higher than in the ZFC state. As the magnetic field is gradually increased towards larger values, the critical current density of the superconductor decreases and the difference between the FC and ZFC state becomes smaller.

To assess the flux penetration in our sample we performed careful $M$-$T$ measurements in the ZFC and FC state for different magnetic fields using a SQUID magnetometer (Fig. 7). The ZFC state is created by cooling the magnetic field after the sample to the base temperature (1.8 K) in zero field. The FC state is created by applying the field at 15 K and cooling the sample to the base temperature in the magnetic field. The $M$-$T$ measurements are carried while warming up the sample from this initial ZFC or FC state. At 10 Oe the ZFC curve shows pronounced diamagnetic response whereas the FC curve is flat and very close to zero as expected for a strongly pinned type-II superconductor. However, as the magnetic field is increased the difference between the FC and ZFC curves progressively decreases, and at 3 kOe the two become indistinguishable within experimental resolution. Thus beyond this field the flux completely enters the ZFC state and the role of flux pinning on the entry of flux in the ZFC state is negligible.

APPENDIX B: COMPARISON BETWEEN ABRIKOSOV VORTEX CORES AND THE VORTEX CORE IN DISORDERED NbN

Here we compare the vortex core in disordered NbN with the Abrikosov vortex core in a clean single crystal of NbSe$_2$ ($T_c \sim 7.2$ K). Figures 8(a) and 8(d) show the images of the vortices acquired at 450 mK in NbN and in a pure NbSe$_2$ crystal, respectively. Figures 8(b) and 8(e) show the normalized tunneling spectra along a line passing through the center of the vortex for the two samples. For NbSe$_2$ we observe that for the normalized tunneling spectrum at the center of the vortex $G(V) \geq 1$ at all biases and the spectrum shows a zero bias conductance peak associated with Caroli–de Gennes–Matricon (CDM) state [36,43] (Fig. 8(f)). In contrast, at the center, the normalized tunneling spectrum for NbN shows a pseudogap [Fig. 8(c)], characterized by a suppression of the coherence peak and a soft gap characteristic of superconducting pairing.
FIG. 8. Comparison of vortex core in NbSe$_2$ and NbN. (a) Conductance map over 200-nm $\times$ 200-nm area of NbN at 40 kOe at fixed bias voltage [$G(V = 2.2 \text{ mV})$]. The vortices are shown as red dots. (b) Normalized tunneling spectra [$(G_N(V)-V)$] along the green line on conductance map in (a) going through the center of one vortex core; the black line corresponds to the spectrum at the vortex core. (c) Representative spectra for NbN at the center of the vortex core (black) and a point away from the core (violet). (d) Conductance map over a 250-nm $\times$ 85-nm area of pure NbSe$_2$ at 7 kOe at fixed bias voltage [$G(V = 1.3 \text{ mV})$]. The dark regions are the vortices with lower values of conductance forming a hexagonal Abrikosov lattice. (e) Tunneling spectra [$(G_N(V)-V)$] along the green line on conductance map in (d) going through one vortex core of NbSe$_2$; dark black line corresponds to the spectrum at the vortex core. (f) Representative spectra for NbSe$_2$ at the center of the vortex core (black) and a point away from the core (violet).

APPENDIX C: CRITERION FOR THE UPPER CRITICAL FIELD FROM TRANSPORT MEASUREMENTS

While determining $H_{c2}(T)$ [or equivalently, $T_c(H)$] from transport measurements different criteria are used in the literature. While sometimes it is defined as the locus of points where the resistance drops to 90% of its normal state value, in other instances it is defined as the locus of points where the resistance falls essentially below the measurable limit. Other criteria, such as 50% of the normal state value, are also occasionally used.

To determine the most appropriate criterion in our context, we measured the diamagnetic shielding response of the superconducting film using a two-coil mutual inductance technique [44,45]. In this technique the superconducting film is sandwiched between a quadrupolar primary coil and a dipolar secondary coil and the mutual inductance ($m$) is measured between the two [Fig. 9(a)]. Below the superconducting transition, the superconducting film partially shields the magnetic field produced by the primary coil from the secondary coil and the real part of the mutual inductance ($m'$) decreases, signaling the onset of the diamagnetic response.

Figure 9(b) shows the $m'-H$ (upper panel) and $R-H$ (lower panel) measured at different temperatures. To measure $m'$ we use an ac excitation field with an amplitude of 3.5 mOe and a frequency of 31 kHz. We observe that the onset of ac shielding response coincides with the field where the resistance drops to 0.05% of its normal state value, below which the resistance drops below our lower measurable limit. Consequently, we define this field as our upper critical field [46]. We conclude that the broad transition region above this field observed in $R-T$ measurements consists of phase fluctuating superconducting puddles where the global superconducting order is destroyed. The same locus of points in the $H-T$ parameter space can also be obtained by measuring $T_c(H)$ from $R-T$ measurements in constant magnetic fields, where $T_c(H)$ is defined as the temperature where the resistance drops to 0.05% of its normal state value [Fig. 9(c)].

APPENDIX D: DETAILS OF NUMERICAL SIMULATIONS

General features of simulations. We consider our two-dimensional $s$-wave superconductor to lie on the xy-plane and represent it as an attractive Hubbard model, and add to it
nonmagnetic random potential on lattice sites as disorder. This is then subjected to an orbital magnetic field, $B_\mathbf{z}$, which is set up by using vector potential in the Landau gauge $A = Bx \hat{y}$. We carry out our numerical simulation of this system on a simulation box of size $L_x \times L_y$ (linear dimension is expressed in terms of the number of the lattice spacing). While the microscopic BdG calculation is performed on a simulation box mentioned above, we repeat such simulation box $m_z$ ($m_y$) times along the $\hat{x}$ ($\hat{y}$) direction, so that the BdG calculations can be performed on each of them, which are then combined through Bloch translation to produce results on a much bigger system of size $m_zL_x \times m_yL_y$ [39]. Note that the idea of repetition of a simulation box for a periodic vortex lattice in a clean system has an exact parallel of such method of repetition of a unit cell in a solid to describe energy band structure. In order to represent a bulk system, we get rid of boundary effects implementing periodic boundary conditions in both $x$ and $y$ directions and allow each simulation box to contain an even number of flux quanta. This is because a superconducting flux quantum has the half strength of a “regular” flux quantum. It is thus the flux that is directly set in our simulation, and not the magnetic field strength. The strength, thus, can be tuned either by changing discretely the (even) number of flux quanta through the simulation box, or by changing the size of the simulation box.

**Chosen parameters.** Using presently available computational resources, BdG simulations for superconductors are restricted to relatively small system sizes. Therefore, the parameters for such simulations have to be carefully chosen so that they represent the actual experimental system and yet the resulting physical quantities such as $\xi$ remain within the size of the simulations. Here we explain the rationale of the chosen parameters in our simulation.

1. **Choice of $|U|$.** A small value ($|U|/t < 1$) is desired for representing a truly weakly coupled $s$-wave superconductor. However, a small $|U|$ makes the coherence length $\xi$ large. Because our numerical resources in the presence of disorder (as discussed below) limit the simulation box to $36 \times 36$ (in the unit of lattice spacing in both $x$ and $y$ directions), we choose $|U| = 1.2t$ which leads to a clean limit coherence length, $\xi_c \sim 10–12$ lattice spacing [38]. In order to track the spatial reorganization, it is important that the linear dimension of our simulation box is at least a few times the coherence length. But we emphasize that a small tuning of $|U|$ does not change any of our qualitative claim. We have checked this by making some test runs for $|U| = 1.5, 2.0$.

2. **Choice of $\rho$.** The choice of $\rho = 0.875$ is also is decided by a trade-off. The strength of superconductivity (say, the magnitude of pairing amplitude) in the clean system is maximum at half filling (i.e., $\rho = 1.0$) for the attractive Hubbard model we study. However, this model, in the absence of disorder, produces a doubly degenerate ground state with ordering in the charge density wave (CDW) channel competing with $s$-wave superconductivity. While disorder wipes out the global CDW, a local and short-ranged CDW ordering persists. In our calculations, we wanted to stay away from such competing orders, as we focused only on the orbital field effects on disordered superconductors. Thus we had to use $\rho < 1$. Pairing amplitude would be very weak even without disorder and magnetic field, if we choose too small a value for $\rho$. The chosen value $\rho = 0.875$ is a reasonable trade-off, and for this same reason disordered superconductivity is widely studied using this value [38,47,48].

3. **Choice of $V$.** The disorder strength, $V$, in our numerical simulation is chosen by matching the width of the distribution of $\tilde{D}_p$ from theory with the distribution of $\tilde{G}_{NP}$ from our experiment. Note that both these distributions will reduce to $\delta$ functions at the corresponding BCS values in the absence of disorder. The width of these (normalized) distributions increases with disorder strength, and thus the width is taken to be the signature for the match of the extent of disorder.

4. **Normalization of the density of states.** Because we carry out our numerics on a lattice, the superconducting DOS does not become flat at energy scales larger than $\Delta$. On the other hand, for a tight binding model the DOS on lattice even without Hubbard attraction is not flat. It is understood that superconductivity arises by opening up a gap in the DOS in a tight binding model at the chemical potential. While this reorganizes the structure of DOS at low $|E|$, it remains unchanged for large $E$. We found that for our model parameters the threshold $|E|$ beyond which the DOS remains unaffected by superconductivity is roughly $0.2t$, and hence it is used for normalization. We avoided normalization by the value of DOS at positive $E$, because of the presence of a close-by Van Hove singularity for our parameters. The singularity falls within the gap for $V = 0$ [see Fig. 5(a)], and does not create trouble, but the situation is more complex in the presence of field with Van

![Graph](image96x659to160x738)

**FIG. 9.** Comparison of magnetic field variation of resistance and diamagnetic shielding response. (a) Schematic diagram of the two-coil mutual inductance setup; the superconducting film is sandwiched between a quadrupolar primary coil and a dipolar secondary coil. (b) Magnetic field variation of $m'$ (upper panel) and resistance (lower panel) at different magnetic fields; the vertical dashed lines show the onset of the diamagnetic shielding response which coincide with the field where the resistance goes below our measurable limit (at 0.05% of its normal state value). The resistance is plotted in log scale for clarity. (c) The loci of $H_c(T)$ [or $T_c(H)$] in the $H$-$T$ parameter space from $m'$-$H$, $R$-$H$, and $R$-$T$ measurements.
Hove singularity and partial gap filling. Thus we renormalize the DOS by its value at $E = -0.2t$.

**Simulation of clean systems $(V = 0)$**. Focusing on a clean system, we choose to work with a rectangular simulation box of size $L \times 2L$ that contains two superconducting flux quanta (leading to two vortices through it). In this case, the magnetic field is tuned by changing $L$. Our construction thus forces a square vortex lattice. This commensurability of the vortex lattice with the underlying lattice on which the electrons live is unavoidable for a simulation box that is not too large, like in the present case. Two vortex centers can appear anywhere in the simulation box due to the translation symmetry, provided the relative distance remains $L$, and we choose them to lie at the center of each square-shaped half simulation box. We start the BdG calculation with guess values of parameters, on which the final self-consistency would be achieved. For example, the guess for the order parameter profile $\Psi(\vec{r})$ is taken by applying the analytical solution of Abrikosov close to $H^2_{c2}$ [49]. This allows a rather accurate starting point for the phase of the order parameter, aiding significantly the convergence of the BdG self-consistency. The procedure outlined here reproduces standard results, e.g., the depletion of pairing amplitude at the vortex core of the diameter $\sim \xi$, expected curling of the phase of the order parameter, and the zero-bias peak at $E/t = 0$ in the local DOS at the vortex core due to the formation of the CDM bound state [36]. To describe our clean system, we use $L = 40$, and $m_x = 20$, $m_y = 10$, which leads to an effective system size of $800 \times 800$. The vortex lattice becomes of size $20 \times 20$, in terms of intervortex spacing. For our clean system $(V = 0)$, we only show the results on the square area through which only one flux quantum passes. These results are shown in Figs. 10(a)–10(c).

**Simulation of disordered systems.** Now moving on to the disordered situation, the lack of translation symmetry in the presence of disorder does not offer a good guess for the local order parameter causing the convergence to self-consistency to be very slow. We used combinations of Anderson, Broyden, and modified Broyden mixing methods to accelerate the convergence to self-consistency [40]. Even then, the number of iterations for self-consistency for $n = 2$ on a given realization of disorder at $V = 0.5$ grows by two orders of magnitude compared to the clean case. As a consequence we cannot simulate a large simulation box unlike the clean case. Thus we use a simulation box of size $36 \times 36$ and increase the magnetic field by changing the number of flux through the simulation box in the presence of disorder. We, however, continue to use the repeated zone scheme [39] with $m_x, m_y = 20$. The vortex lattice being disordered, the periodic repetition in this case does not add any additional physical meaning as for the clean periodic vortex lattice, but offers an enhanced resolution of our results, for example in DOS in Figs. 5(a) and 6(k) (inset). Confidence in our results was developed by choosing different initial conditions and arriving at the same final self-consistent value of the complex order parameter. Because we already use a large simulation box and the self-consistency is numerically expensive, we tune the magnetic field only by changing the number of flux quanta ($n = 2, 4, 6, \ldots$) through the simulation box. This has a limitation that the field strength can alter only

![FIG. 10. Vortex simulation in a clean superconductor. (a) Surface plot of the pairing amplitude $|\Psi|$ on a scale of 0 to 1 in a square simulation box illustrating the vortex. (b) Phase of the superconducting order parameter in the simulation box. (c) Local density of states for the vortex core region showing the CDM peak (black) and at regions away from the vortex core region (blue).](image-url)
in large discrete steps. A change of magnetic field in small steps would require a simultaneous change of the system size, which is computationally much more expensive and beyond the scope of the present work.


[46] This definition differs from the 90% criterion used in Ref. [28], where the $H_c^2$ was determined from transport measurements alone.

