Application of a Fourier transform based filtering technique to improve signal-to-noise ratio in modulation spectroscopy experiments

Sandip Ghosh[a)] and B. M. Arora
Solid State Electronics Group, Tata Institute of Fundamental Research, Colaba, Mumbai 400 005, India

(Received 7 April 1997; accepted for publication 12 May 1997)

It is shown that a simple rectangular filter is a good approximation to Wiener optimal filter for improving the signal-to-noise ratio in a typical modulated reflectance spectroscopy experiment. This enables one to obtain identical information from data recorded with much smaller time constants for data averaging and therefore faster scan speeds. The limitations of this method are also discussed. © *1997 American Institute of Physics.* [S0034-6748(97)02908-0]

In recent years, certain modulated reflectance spectroscopic techniques such as photoreflectance (PR) and contactless electroreflectance (CER) have gained considerable popularity as nondestructive tools for studying semiconductor heterostructures and devices.[1–3] In a typical experiment,[1,4] the signal which is proportional to the change in the reflectance of the sample (ΔR) is much smaller as compared to the background proportional to its reflectance (R). Typically $\Delta R/R \simeq 10^{-4}-10^{-6}$, therefore requiring phase sensitive detection using a lock-in amplifier. However, electronics and small fluctuations in the pump/probe beam intensity among other causes result in noise, whose measurement and reduction has received considerable attention.[5,6] To improve the signal-to-noise (S/N) ratio one normally uses large time constants for data averaging, hence very slow scan speeds. This is disadvantageous for routine application of these techniques as characterization tools. We describe below the use of a fast Fourier transform (FFT) based simplified filtering procedure to improve S/N ratio, thereby enabling faster data acquisition.

Figure 1(a)–1(i) shows a typical PR spectrum, the sample being a GaAs/$\mathrm{In}_x\mathrm{Ga}_{1-x}\mathrm{As}$ single quantum well. Figure 1(b) shows the logarithm of the power spectrum of the above data, wherein the logarithm of the square of the modulus of $\mathscr{C}(f)$ is plotted as a function of frequency. $\mathscr{C}(f)$ is the Fourier transform of the above data. The frequency scale has been normalized with respect to the Nyquist frequency (f_c). Standard FFT routine[7] was used for obtaining $\mathscr{C}(f)$. The power spectrum typically has a prominent hump at smaller frequencies which correspond to the actual signal in the PR spectrum. At a certain frequency f_o there is a sudden change of slope and beyond it lies a jagged monotonically decreasing background. These higher frequencies correspond to the noise in the PR spectrum. The principal of noise reduction is based on applying a suitable filter to reduce the amplitudes at frequencies corresponding to the noise in the Fourier transformed data and then performing an inverse transform to get back the original spectrum with reduced noise. In the present case the appropriate filter to use is Wiener optimal filter whose transmission coefficient is given by $T(f)=|s(f)|^2/(|s(f)|^2+|n(f)|^2)$, where $|s(f)|^2$ and $|n(f)|^2$ represent the signal and the noise strengths in the power spectrum, respectively. It is constructed[7] by fitting two different smooth curves through the signal and the noise dominated regions in the power spectrum to estimate $|s(f)|^2$ and $|n(f)|^2$, respectively, in these regions. Thereafter these curves are extrapolated to the rest of the frequency spectrum and the above relation is used to determine $T(f)$. Figure 1(c) shows the Wiener filter constructed using this procedure for the power spectrum in Fig. 1(b). The near rectangular shape (with rounded edges) of the Wiener filter is found to be typical, which prompted us to consider instead a rectangular filter [$T(f)=1$ for $f<f_o$ and $T(f)=0$ for $f>f_o$], which is far easier to construct. The PR spectrum in Fig. 1(a)–1(i) filtered with the rectangular filter is shown in Fig. 1(a)–1(ii). It is seen that the spectral features in the former are identically reproduced in the latter. We find no appreciable difference between the PR spectrum filtered with the Wiener filter and that with the approximate rectangular filter. Also the line shape functions[1] fitted to the main features in Fig. 1(a)–1(i) and (ii) give identical values of the fitted parameters.

Next we demonstrate an application of this technique to a spectrum with poor S/N ratio. For practical purposes we define a quantity which we shall call detection resolution (Λ) given by $\Lambda = 3\mathscr{S}\tau$, where $\mathscr{S}$ is the scan speed (nm/min) and τ is the time constant setting (min). To prevent the spectral line shape from being distorted by the temporal response of the detection system, Λ must be kept much smaller ($\simeq$ 1/5) than the band pass ($\Delta\lambda$) of the monochromator. Figure 2(i) shows a CER measurement on a $\mathrm{GaAs}_{1-x}\mathrm{P}_x$ alloy which was taken with a relatively high scan speed of 30 nm/min, with $\Delta\lambda = 2.5$ nm. To keep Λ much smaller than $\Delta\lambda$, τ was set to 100 ms so that Λ was $\simeq 0.2$ nm. The small time constant resulted in increased noise in the $\Delta R/R$ spectrum, its root mean squared (rms) value being 13×10^{-7}. Figure 2(ii) shows the same spectrum after being filtered using a rectangular filter as described above. In this spectrum, the rms noise is down to 2.5×10^{-7}, a factor of 5 improvement. The filtered spectrum also indicates an additional feature at 887 nm which was not clearly evident in the unfiltered spectrum. Figure 2(iii) shows the spectrum of the same sample taken with a much longer τ (3 s) and therefore much smaller $\mathscr{S}$ (3.8 nm/min) to achieve a small Λ (0.6 nm) value. The rms noise in this spectrum is 3.0×10^{-7}. Fitting Aspnes' third derivative functional form line shape function[1] to the main feature in Fig. 2 (ii) and (iii) we get identical values for the band gap (1.450 eV) and phenomenological

---

[a)] Electronic mail: sangho@tifrvax.tifr.res.in

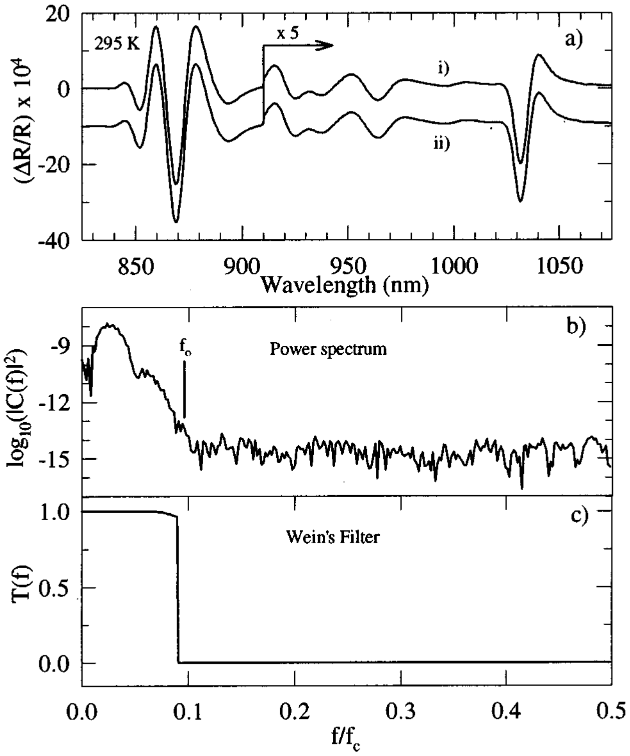

FIG. 1. (a) (i) PR spectrum of a GaAs/$In_xGa_{1-x}As$ single quantum well. (ii) The same spectrum after being filtered using a rectangular filter. Plots are vertically shifted for clarity. (b) The Fourier power spectrum of the PR spectrum in (a)–(i). (c) The Wiener filter constructed using the power spectrum in (b).

broadening parameter (14.5 meV) from the two spectra. Figure 2(iii) also indicates that the additional feature seen at 887 nm in the filtered spectrum is genuine and probably originates from the GaAs substrate underneath. Thus we have been able to speed up the measurement by a factor of 8 without the loss of any spectral information while achieving a significant improvement in the S/N ratio.

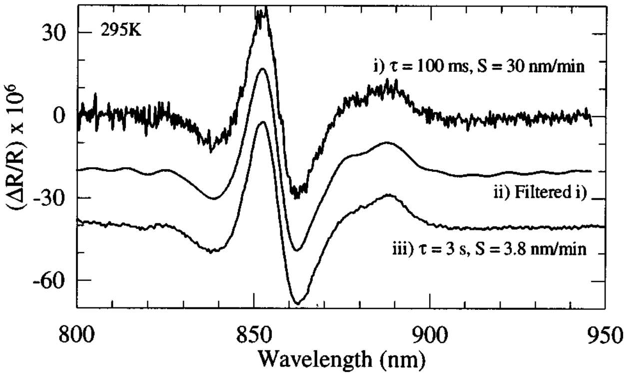

FIG. 2. (i) CER spectrum of a $GaAs_{1-x}P_x$ alloy. (ii) The same spectrum after being filtered. (iii) The CER spectrum measured with a higher time constant (τ) and smaller scan speed ($\mathscr{S}$). Plots are vertically shifted for clarity.

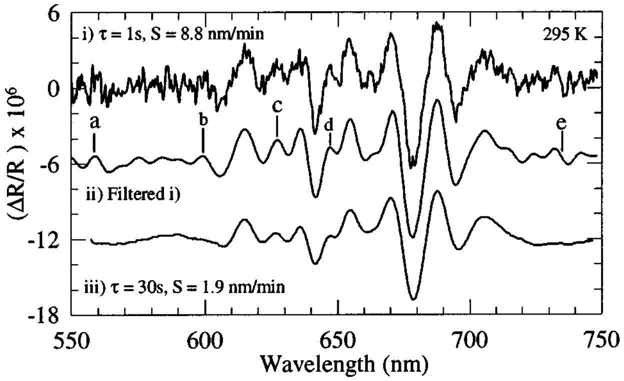

FIG. 3. (i) CER spectrum of a GaP/InP superlattice on a GaAs. (ii) The same spectrum after being filtered. (iii) The CER spectrum measured with a higher time constant (τ) and smaller scan speed ($\mathscr{S}$). Plots are vertically shifted for clarity.

However this technique has the following drawback. Notice that in Fig. 1(b), the part of the noise spectrum which has the same frequency spread as the actual signal is not removed in the filtering process. This can cause serious problems if, to begin with, the S/N ratio is very poor as illustrated in the following example. Figure 3(i) shows the CER spectrum of a GaP/InP superlattice on a GaAs taken with $\tau=1$ s and $\mathscr{S}=8.8$ nm/min ($\Lambda \simeq 0.45$ nm). Figure 3(ii) shows the same spectrum after filtering while Fig. 3(iii) shows its spectrum taken with $\tau=30$ s and $\mathscr{S}=1.9$ nm/min ($\Lambda \simeq 2.9$ nm). Comparing the latter two we see that the large Λ value has resulted in the features getting stunted and poorly resolved in Fig. 3(iii). But more importantly, the features marked (c) and (d) in Fig. 3(ii) are present in Fig. 3(iii) while those marked (a), (b), and (e), although comparable in magnitude to (c) and (d), are absent. These spurious features (a), (b), and (e), arise due to unfiltered noise which has the same frequency spread as the actual signal, and since the signal level is very weak, these can easily be mistaken for real. A proper determination of the S/N ratio, for which this filtering technique may lead to the above problem, requires computation of the relative area under the signal and noise dominated regions in the power spectrum. However, as a rule of thumb, if the power at peak of the signal dominated region in the power spectrum is at least three orders of magnitude more than the power at f_o then this technique can be safely applied.

In conclusion we have demonstrated that a simplified Fourier filtering procedure can be used to improve S/N ratio in a modulation spectroscopy experiment, and thereby help decrease the data acquisition time considerably. The authors thank Professor K. L. Narsimhan for his encouragement and Professor Hajime Asahi for providing the superlattice sample.

[1] F. H. Pollak and H. Shen, Mater. Sci. Eng. Rep. **R-10**, 275 (1993).
[2] H. Shen and M. Dutta, J. Appl. Phys. **78**, 2151 (1995).
[3] F. H. Pollak, W. Krystek, M. Leibovitch, M. L. Gray, and W. S. Hobson, IEEE J. Sel. Topic. Quantum Electron. **1**, 1002 (1995).
[4] S. Ghosh and B. M. Arora, *Advances in Instrumentation, Proceedings of the International Conference on Instrumentation* (New Age, Bangalore, 1996).
[5] D. Y. Dan, H. Qiang, and F. H. Pollak, Rev. Sci. Instrum. **65**, 1988 (1994).
[6] H. Shen and M. Dutta, Appl. Phys. Lett. **57**, 587 (1990).
[7] W. H. Press, S.A. Teukolsky, W. T. Vetterling, and B. P. Flannery, *Numerical Recipes*, 2nd ed. (Cambridge, New York, 1992).