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ISSUES IN THE DESIGN OF A DEDICATED FUZZY CONTROLLER FOR AUTOMATIC PROCESS CONTROL

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ABSTRACT:

This paper rationalizes the use of Fuzzy Logic in Automatic Process Control. The abstractness of Fuzzy Logic Principles is masticated into numerical and logical manipulations that are easily digestible by today's digital processing circuits. The large number of simple floating-point operations that result, suggest the possibility of the design of a dedicated Fuzzy controller whose features are discussed. An analytical approach is taken to circumvent implementation problems whilst still preserving the basic concepts of Fuzzy Logic.

KEYWORDS:

Fuzzy Logic, Membership Value, Fuzzy Curve, Membership Function, Scaling Factor, Parabolic Section, Centroid, Intersection Area, Simpson's Rule.

1. INTRODUCTION:

In recent times, the relevance of Fuzzy Logic in Process Control has been established in both theory and practice. The key difference between traditional Automatic Process Control and Fuzzy Logic-based control is that in the former, a mathematical model of the process being controlled is arrived at and becomes the basis while in the latter, the focus is on the control rules being as expressed by a human expert. These control rules employ many common Linguistic (Fuzzy) terms like 'low', 'high' etc.. The controller can interpret these otherwise inexpressible terms using a methodology that is based on the concepts of Fuzzy Logic.

As opposed to ordinary two-valued logic, in Fuzzy Logic all variables have their values mapped elastically into a curve representing the Linguistic term associated with the variable and the result is a gradation called the Membership Value of the variable value in the range [0,1] of the curve.

The advantage of using Fuzzy Logic for Process Control lies in the fact that fewer control rules are needed and that since all the control rules are simultaneously involved in the determination of the final control output, the resulting control output is smoothly regulated and the system as a whole is rendered more robust and fault-tolerant. Also, once the Linguistic terms involved in the control rules are identified clearly, the corresponding Membership Curves can be derived empirically in a way that is independent of the Process Control environment and this means that the rule-base can be created independent of the actual values of the inputs to and the outputs of the controller - only the relation between the two is required to be known.

2. Mathematical Concepts Related To Fuzzy Process Control:

2.1 Definitions And Terms:

The application of Fuzzy Logic techniques to Automatic Process Control is tied together with the concept of Linguistic Control rules [HoOs82]. The control strategy is rule-based and comprises of rules of the structure :

IF <CONDITION-GROUP> **THEN** <CONTROL-ACTION-GROUP> (I)

where the process-control variables that influence the process appear in the CONDITION-GROUP and the results of their variables being controlled appear in the CONTROL-ACTION-GROUP. Each process-control variable and each control-action variable is associated with a Fuzzy Set. The elements of the Fuzzy Set are Fuzzy Membership Functions representing the Linguistic Terms that describe either the process-control variable or the control-action variable accordingly. It is the Fuzzy Linguistic Terms that actually appear in the Groups in (I). If a group has more than one Fuzzy Membership Function, then they are linked by logical operators.

2.2 Some Typical Parameters for Control Rule Synthesis:

LINGUISTIC TERMS - HIGH, LOW, OK ,MPOV ...

LOGICAL OPERATORS - AND, OR, NOT ...

PROCESS-CONTROL and/or CONTROL-ACTION VARIABLES

- TEMP (Temperature),
- △TORQ (Change in Torque),
- △FUEL (Change in Fuel Quantity) ...

CONDITION-GROUP - HIGH(TEMP) AND LOW(△TORQ) ...

CONTROL-ACTION-GROUP - MPOV(△FUEL) ...

CONTROL RULE -

IF HIGH(TEMP) AND LOW(ΔTORQ) THEN MPOV(ΔFUEL) .

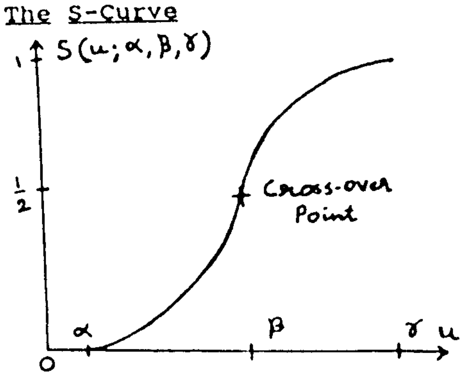
The Fuzzy Membership Functions are implemented by continuous curves in the interval [-1,1]. These curves are obtained empirically and are drawn from the family of smooth exponential and/or parabolic curves. For each of these curves, the abscissa corresponds to a suitably scaled-down value of the process control variable and the ordinate corresponds to the Membership Value or Grade of Membership of that variable in the Membership Function.

All Membership Functions have a range of [0,1] and a domain of [-1,1].

2.3 S-Curves and π-Curves:

The following are some general equations for the Fuzzy Membership Function Curves [Za75]:

The S-Curve



$S(u;\alpha,\beta,\Gamma) = 0$ for $u \le \alpha$

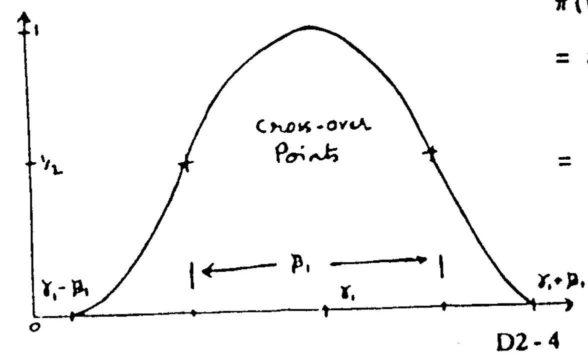
$= 2*(u-\alpha)^2/(\Gamma-\alpha)^2$ for $\alpha \le u \le \beta$

$= 1 - 2*(u-\Gamma)^2/(\Gamma-\alpha)^2$ for $\beta \le u \le \Gamma$

$= 1$ for $u \ge \Gamma$

$\beta = (\alpha+\Gamma)/2$

The π-Curve



$\pi(u;\alpha_1,\beta_1,\Gamma_1)$

$= S(u;\Gamma_1-\beta_1,\Gamma_1-\beta_1/2,\Gamma_1)$ for $u \le \Gamma_1$

$= 1 - S(u;\Gamma_1,\Gamma_1+\beta_1/2,\Gamma_1+\beta_1)$ for $u \ge \Gamma_1$

The parameters α, β, Γ, α_1, β_1, Γ_1 are chosen empirically so that different smooth curves are obtained to represent different Fuzzy Linguistic Terms. Both, the S and the π Curves are composed of Parabolic Curves and are deliberately chosen to simplify computation later on.

2.4 Logical Operators:

The logical operators are defined as follows:

$$f \text{ AND } g \equiv \min(f,g)$$

$$f \text{ OR } g \equiv \max(f,g)$$

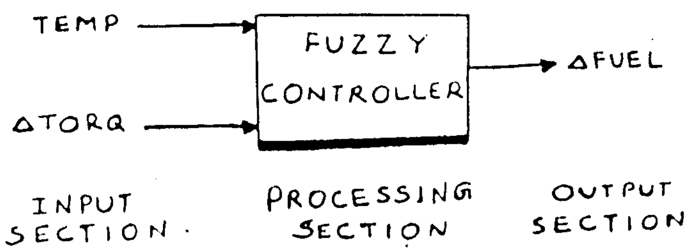
$$\text{NOT } f \equiv 1 - f$$

where f, g are Membership Values (ordinates) obtained from the Fuzzy Curves.

3. Methodology:

3.1 An Example:

Let us assume that we have a single control action variable **FUEL** (change in Fuel Quantity) which is controlled by two process control variables **TORQ** (change in Torque) and **TEMP** (Temperature) [HoOs82].



3.1.1 Assignment of Equations to Linguistic Terms:

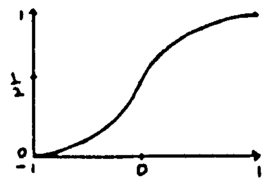
The Fuzzy Membership Functions involved are shown in Table 1 along with their corresponding equations, derived from the S- and π-curves.

Table 1

(1) **HIGH** (High): $\alpha = -1$, $\beta = 0$, $\Gamma = 1$

$y = \frac{1}{2}(x+1)^2$ for $-1 \le x \le 0$

$= 1 - \frac{1}{2}(x-1)^2$ for $0 \le x \le 1$

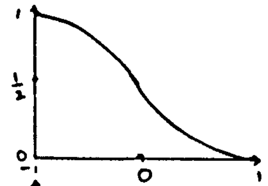


HIGH

(2) **LOW** (Low): $\alpha = 1$, $\beta = 0$, $\Gamma = -1$

$y = 1 - \frac{1}{2}(x+1)^2$ for $-1 \le x \le 0$

$= \frac{1}{2}(x-1)^2$ for $0 \le x \le 1$



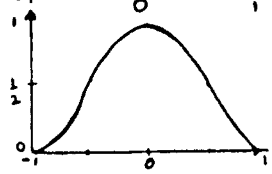
LOW

(3) **OK** (Okay): **= LOW + HIGH**

$y = 2(x+1)^2$ for $-1 \le x \le -\frac{1}{2}$

$= 1 - 2x^2$ for $-\frac{1}{2} \le x \le \frac{1}{2}$.

$= 2(x-1)^2$ for $\frac{1}{2} \le x \le 1$



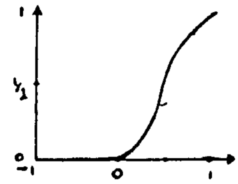
OK

(4) **POV** (Positive): $\alpha = 0$, $\beta = \frac{1}{2}$, $\Gamma = 1$

$y = 0$ for $-1 \le x \le 0$

$= 2x^2$ for $0 \le x \le \frac{1}{2}$

$= 1 - 2(x-1)^2$ for $\frac{1}{2} \le x \le 1$



POV

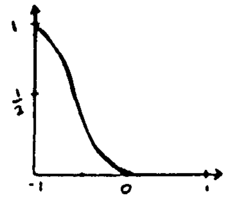
(5) **NEV** (Negative): $\alpha = 0$, $\beta = -\frac{1}{2}$, $\Gamma = -1$

$y = 1$ for $x \le -1$

$= 1 - 2(x+1)^2$ for $-1 \le x \le -\frac{1}{2}$

$= 2x^2$ for $-\frac{1}{2} \le x \le 0$

$= 0$ for $x \ge 0$



NEV

(6) **ZERO** (Zero): **Same as** OK.

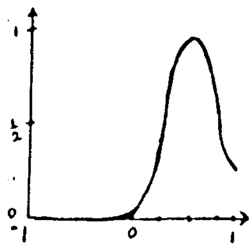
(7) **MPOV** (Medium Positive):

$y = 0$ for $x \le 0$

$= 8x^2$ for $0 \le x \le \frac{1}{2}$

$= 1 - 8(x-\frac{1}{2})^2$ for $\frac{1}{2} \le x \le 0.75$

$= 8(x-1)^2$ for $0.75 \le x \le 1$



MPOV

(8) **MNEV** (Medium Negative):

Put -x in place of x in the equations for MPOV above.

3.2 The Rule Base and Rule-Firing:

Given below is the set of control rules used in our example -

<u>Rule 1</u> **IF ΔTORQ IS ZERO AND TEMP IS LOW THEN MNEV ΔFUEL**

<u>Rule 2</u> **IF ΔTORQ IS POV AND TEMP IS OK THEN MPOV ΔFUEL**

<u>Rule 3</u> **IF ΔTORQ IS NEV AND TEMP IS HIGH THEN ZERO ΔFUEL**

ΔTORQ is associated with Linguistic terms **ZERO, POV, NEV; TEMP** is associated with Linguistic terms **LOW, OK, HIGH** and **ΔFUEL** is associated with Linguistic terms **ZERO, MPOV** and **MNEV**.

The values for **ΔTORQ** and **TEMP** are input from the external world and scaled-down into the [-1,1] interval. The scaled-down values form the abscissae which are used to obtain ordinates (Membership values) from the equations of the curves. Every rule contributes one ordinate for every control variable. In our example, two sets of three ordinates are obtained. Then, the logical operators from the rule-base are applied to individual pairs comprising of two ordinates - one from each set. Here, the ANDing logical operation selects the minimum of the two ordinates for each rule and we end up with three minimal ordinates which are now to be mapped into the Fuzzy Membership curves associated with **ΔFUEL**. This essentially involves scaling down the equation of the curve of the control action variable **ΔFUEL** by a scaling factor equal to the ordinate (whose value lies in [0,1]) that resulted from the preceding logical operation. It leads to a new set of scaled-down equations for the Fuzzy functions associated with **ΔFUEL**. Note that this scaling is only along the Y-axis (i.e. along the Membership Value axis) while the curve remains unchanged along the X-axis (i.e. the Process variable axis).

3.3 Arriving at the Final Control Action:

Once the Fuzzy functions associated with the control variable have been scaled-down using their corresponding scaling-factors, the areas under these scaled-down curves are ORed i.e. superimposed - this superimposition signifies the involvement of all the rules in the rule-base in arriving at any control output. The final control action is then the abscissa of that point in the superimposed area which represents the superimposed area most completely. The centroid of any planar area is the best representative of that area; hence, the final control action that we desire is the abscissa of the centroid of the superimposed area.

In order to obtain the centroid of the superimposed area, we consider the following:

<1> Each Fuzzy function curve is made up of one or more parabolic sections with or without X-axis segments or parallel line-segments.

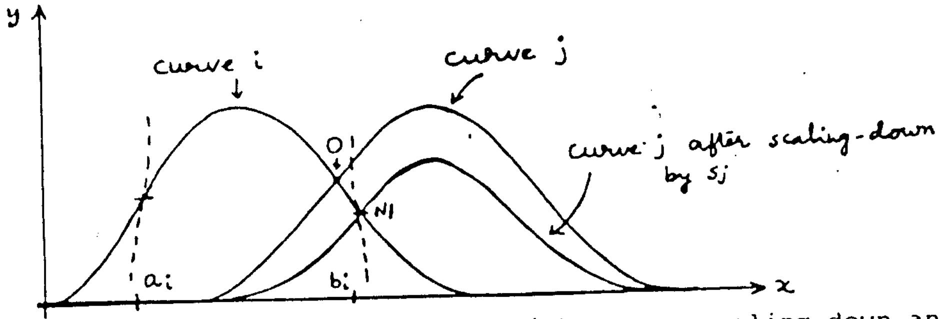
<2> When two Fuzzy curves intersect, we note that it is basically an intersection of parabolic sections.

<3> The information database associated with the physical representation of the Fuzzy curves also contains information about the coordinates of the points of intersection of each curve with every other curve - in the absence of any scaling-down (i.e. when the scaling factor is 1).

<4> When a scaling operation occurs on the intersecting curves, the point of intersection slides. The new point of intersection is required in the calculation of the intersecting area and its coordinates can be derived analytically, given the coordinates of

the original (unscaled) intersection point, the parameters of the parabolic sections (basically the vertices and lengths of latus rectums), and the scaling-factors for the two intersecting Fuzzy curves.

<5> A case may arise wherein, the scaling-down forces the intersection point to slide into the adjoining parabolic section of the Fuzzy curve i.e. beyond the cross-over point of that curve. When this happens, the parameters of the corresponding parabolic section should be considered for deducing the coordinates of the new intersection point. However, we still have to be able to detect when this sliding-over occurs and this can be done as shown:



O is the old intersection point without any scaling-down and N1 is the intersection point with only curve j being scaled-down by a scaling-factor S_j ($0 \le S_j \le 1$). Also, assume that N1 is on the crossover-point for curve i.

Let $O \equiv (t_x, t_y)$ and $N1 \equiv (x_{t1}, y_{t1})$.

Now, both O and N1 must satisfy the equation of the curve i. Hence,

$$(t_x - h_i)^2 = 4p_i(y_t - k_i) \quad \text{(II)}$$

and,

$$(x_{t1} - h_i)^2 = 4p_i(y_{t1} - k_i) \quad \text{(III)}$$

Subtracting (III) from (II) and using $y_{t1} = s_j . t_y$,

$$(t_x - x_{t1})(t_x + x_{t1} - 2h_i) = 4p_i(t_y - y_{t1})$$

$$\Longrightarrow \quad x_{t1}^2 - 2h_i x_{t1} + 2h_i t_x - t_x^2 + 4p_i t_y (1 - s_j)$$

$$\Longrightarrow \quad x_{t1} = ((h_i \pm ((h_i - t_x)^2 - 4p_i t_y (1 - s_j^*))^{1/2})/2 \qquad \text{(IV)}$$

Let a_i and b_i be the end-points of the interval for which curve i is represented by the parabola with vertex at (h_i, k_i) and latus rectum $4p_i$. Then N1 being the cross-over point, has its abscissa equal to b_i. Hence, from (IV), with $s_j \equiv s_j^*$,

$$2b_i = (h_i \pm ((h_i - t_x)^2 - 4p_i t_y (1 - s_j^*))^{1/2})/2$$

$$\Longrightarrow \quad s_j^* = 1 + ((2b_i - h_i)^2 - (h_i - t_x)^2)/(4p_i t_y) \qquad \text{(V)}$$

s_j^* is the threshold value for the scaling factor of curve j.

When $s_j < s_j^*$, the point O slides over beyond the cross-over point into the parabolic section of curve i that begins at interval point b_i and becomes N1 whose coordinates can be obtained using (IV) - the values for h_i, k_i and p_i in (IV) are then the parameters of the corresponding parabolic section. In case the point O is located in a section that has one parabolic section on either side, the choice of the adjacent section can be made by computing slope of curve i at N1 (at cross-over point) in terms of scaling factor and then using the sign of the slope. Similarly, if $s_j > s_j^*$ then O slides over to N1 which is located in the same parabolic section as O and whose parameters must then be used in (IV) in order to calculate the coordinates of N1.

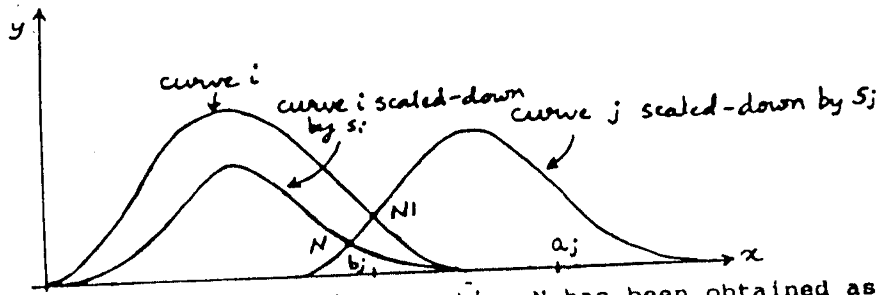
It must be noted that O must be calculated apriori and along with its coordinates, the parameters of the parabolic section carrying O must also be made available.

<6> Once the point N1 has been determined from (IV) after ascertaining whether $s_j < s_j^*$ or not from (V), we keep curve j fixed

at its new scaled position and scale curve i by s_i and proceed to evaluate the actual intersection point N similarly, using (VI) and (VII) below:

$$x_{t2} = ((h_i \pm ((h_i - t_x)^2 - 4p_i t_y (1 - s_j^*))^{\frac{1}{2}})/2 \quad \text{(VI)}$$

$$s_i^* = 1 + ((2b_j - h_j)^2 - (h_j - t_x)^2)/(4p_j t_y) \quad \text{(VII)}$$



<7> Once the point of intersection N has been obtained as above for each pair of curves i,j (i <> j, i < j), we can obtain the abscissa of the Centroid of the superimposed area using the Centroid formula:

$$c_x = \frac{c_{x1}A_1 + c_{x2}A_2 + \ldots + c_{xn}A_n - \Sigma\Sigma c_{xij}A_{ij}}{A_1 + A_2 + \ldots + A_n - \Sigma\Sigma A_{ij}} \quad \text{(VIII)}$$

where c_{xk} represents the abscissa of the centoid of area A_k under the curve k. In (VIII), the term $\Sigma\Sigma c_{xij}A_{ij}$ represents the nett overlapping area as a result of the intersection between various curves. c_{xij} is the abscissa of the centroid of the intersection area between curves i and j (A_{ij}) and is computed as:

$$c_{xij} = \frac{-i\int^{x_i} x.f(x)\,dx}{A_{xij}}$$

where x_i is the abscissa of the point of intersection, obtained as explained earlier.

In case there are more points of intersection between the curves i and j, they can all be computed similarly and absorbed with A_{ij} changing sign appropriately.

C_x, when suitably scaled-up, represents the final control action to be output to the device being controlled.

4. Dedicated Fuzzy Controller Design Issues:

Having taken a look at how Fuzzy Process Control works, we can now proceed to discuss the various issues involved in the design of a Dedicated Fuzzy Controller/Processor.

4.1 Justification:

The entire procedure for employing Fuzzy Logic in Process Control is computationally intensive and involves a large number of simple floating-point operations. Since the outputs(control actions) have to be obtained in real-time in most cases, a dedicated hardware implementation of the mathematical and logical operations involved is necessary.

Also, there are many situations in industry where machines cannot be employed for process control due to their jerky actions resulting from application of Boolean Logic principles - for example, silk garments can't be woven by mechanized (CNC) looms as their abrupt movements cause impusive tension which leads to snapping of the delicate silk threads used.

Dedicated Fuzzy controllers could control such machines as their output control actions vary more smoothly and gradually; this being made possible by the fact that all the rules in the rule-base are considered for every output control action that results from the input process control variables. Also, the Linguistic terms used are represented by smoothly varying curves.

Moreover, Automatic control in consumer products (Televisions, Washing machines etc..) and in specialized equipment (Nuclear Reactors, Oil Rigs etc..) demands the use of dedicated controllers as high reliability is expected and human involvement may not be efficient enough or may even be impossible.

4.2 Feasibility of Practical Implementation:

The results ofconversion of abstract principles of Fuzzy Logic to numerical and logical manipulations using analytical methods are extemely suitable for rendition into hardware logic.

The entire computation can be seen to consist of only the following basic floating-point operations:

FCOMP - Floating-point comparision.
FADD - Floating-point addition.
FSUB - Floating-point subtraction.
FMUL - Floating-point multiplication.
FDIV - Floating-point division.
FSQRT - Floating-point square-root.

4.2.1 The Lookup-table:

In addition, the lookup-table necessary for most parts of the computation procedure can be implemented as follows:

-1	a_1	a_2	. . . . . .	a_{n-1}	1	: Data
0	1	2	. . . . . .	n-1	n.	: Offsets

The above table contains the values that represent the starting-points of the various adjacent intervals corresponding to parabolic sections that make up the smooth Fuzzy curves.The table can be used to lookup the section a given abscissa value belongs to by comparing it with each of the values stored in the table and locating the first table-value that is greater than the given value. Then, the offset of that table-value can be used to vector to a routine that does various computations for the parabolic section that occurs in the interval for which the located table-value is the end-point. There could be a seperate set of routines for each section of the Fuzzy curve and there could be seperate sets for each type of computation required. For example, sets of routines could:

a> obtain ordinates (membership-values) corresponding to the given abscissa value.

b> obtain areas under the curve upto the given abscissa value.

c> Obtain centroids etc...

4.3 Accuracy, Speed and Efficiency:

1> Even though the analytical methods used are computationally intensive, a lot of information in the information-base of the controller can be calculated apriori and independent of the application for which the controller is designed; given that the Fuzzy rules and curves have been decided upon. This information-base can be hard-coded into the controller's micro-code.

2> The total number of computations in the evaluation of the nett area of intersection in (VIII) is polynomial and is given as n^2-3n for n rules in the rule-base and assuming any two curves intersect in only one point.

3> The accuracy of the floating-point values encountered now depends more on the available technology than on the methodology as the parabolic sections considered ensure that only square-roots and multiplications involving two terms at a time are required. Also, Simpson's rule [JoKi78] needs only the end-points and the mid-point of the parabolic section considered. Small inaccuracies can creep in only when cubic expressions are encountered in the computation of centroids and when there are more than two points of intersection between any two Fuzzy curves.

4> In addition, the normal computation problems associated with floating-point operations [By88] are to be taken into account and the analytical results (IV), (V), (VI), (VII) and (VIII) should be rewritten/regrouped accordingly.

4.4 Cost and Compatibility:

Since most of the basic floating-point operations desired can be implemented using circuits employing existing technology, the overhead resulting from the use of a new methodology is not much. Besides, better performance can be expected from machines controlled by Fuzzy principles.

In order to keep compatibility with existing computing systems, it is suggested that the Fuzzy controller be made available in the form of a dedicated/intelligent coprocessor.

5. Extensions and Improvements:

Errors resulting from extensive Floating-point computation involved in the analytical solutions presented can be reduced by iterative techniques which, though time-consuming, converge to more accurate results.

It is obvious from the analysis presented that Fuzzy Process Control exhibits an inherent parallelism at both task level and sub-task level. This can be exploited by employing a parallel architecture for the design of the Fuzzy controller.

6. Conclusion:

The methodology evolved in this paper has been targetted at practical implementation. This inturn dictates that at every stage of computation, there should be as few unknowns as possible and it has been achieved to a large extent by expressing everything in terms of the scaling-factor which is then obtained dynamically from the inputs to the system.

7. Acknowledgements :

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