Throughput-Optimal Broadcast in Wireless Networks with Point-to-Multipoint Transmissions

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Outline

Motivation

- We consider the problem of optimally broadcasting packets in a multi-hop wireless adhoc network

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- We consider the problem of optimally broadcasting packets in a multi-hop wireless adhoc network
- A primary measure of efficiency is *throughput-optimality*, i.e., policies that achieve the entire capacity region
- Vast literature for the Unicast problem (Backpressure policy), not so much for other flow problems
  - Packet duplications are harder to deal with (no flow conservations)

Introduction

- We study the Generalized Flow Problem and design throughput-optimal policies.
- A *Fundamental* problem with wide ranging applications: Internet routing, in-network function computations, live multi-media streaming, military communications etc.
- Topics of this talk:

  1. Broadcast: Specialized dynamic algorithms that solve the throughput-optimal broadcasting problem
     - It admits an inherently decentralized solution

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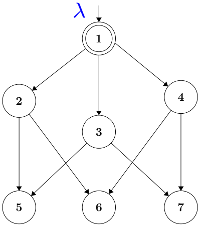
- Topics of this talk:
  1. Broadcast: Specialized dynamic algorithms that solve the throughput-optimal broadcasting problem
     - It admits an inherently decentralized solution
  2. Generalized Flow: A general algorithmic paradigm that efficiently solves *all flow problems* (unicast+broadcast+multicast+anycast).

System Model

- The Wireless Network is represented by a graph $\mathcal{G}(V, E)$, where each node has an omnidirectional antenna.
- Packets arrive at a source node r i.i.d. at every slot at rate λ.
- Due to the *local broadcast* nature of the wireless medium, packets transmitted by a node i is heard at all of its out-neighbor $j \in \partial^+(i)$.
- As a result, if two or more in-neighbors of a node transmits at a slot, it results in a collision.
- This talk considers collision-free schedules only. The set of all feasible collision-free node activations is given by $\mathcal{M}$.

An Illustrative Example



A wireless network

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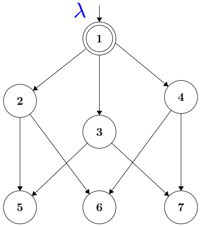

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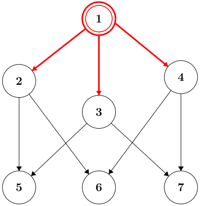

Activation $\boldsymbol{s}_1$

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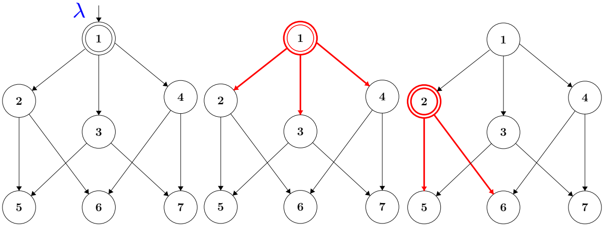

A wireless network | *Activation* s_1 | *Activation* s_2

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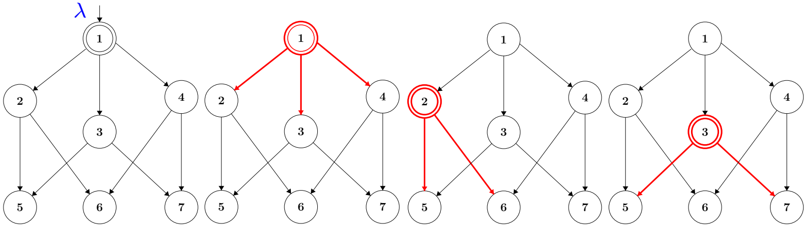

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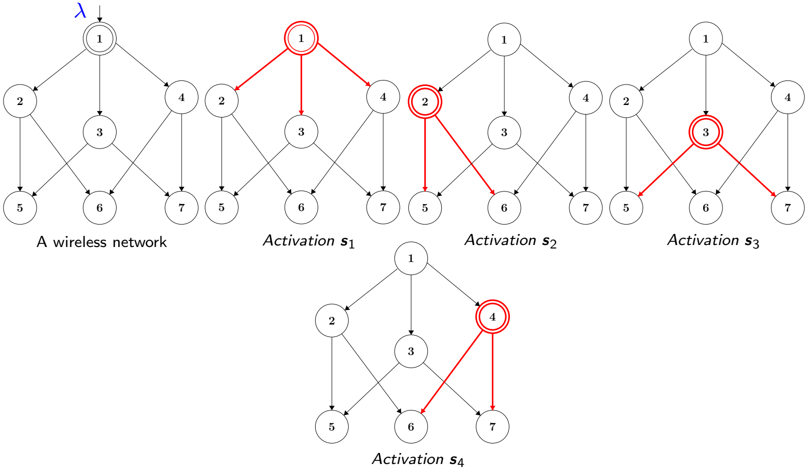

A wireless network and its feasible link activations under the primary interference constraints.
$\mathcal{M} = \{s_1, s_2, s_3, s_4\}$

Wireless Broadcast: Problem Formulation

A feasible broadcast policy $\pi \in \Pi$ executes the following two actions at every slot t :

- Node Activation $\pi(\mathcal{A})$: Activates a subset of nodes $\boldsymbol{s}(t) \in \mathcal{M}$ subject to the underlying interference constraints.
- **Packet Scheduling $\pi(\mathcal{S})$:** The activated nodes *locally broadcasts* a set of packets subject to the capacity/power constraints of the nodes.

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- Let $R^\pi(T)$ denote the number of packets received in common by all nodes under the action of a broadcast policy π.
- The objective is to design a policy π such that for all $\lambda < \lambda^*$

$$\liminf_{T\to\infty} \frac{R^\pi(T)}{T} = \lambda, \quad \text{w.p. } 1,$$

where λ^* is the broadcast capacity of the network.

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Hardness Result

Our first result in this point-to-multipoint broadcast setting is the following:

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- This is surprising, because we showed earlier [Sinha et al, 2015, 2016] that the problem is efficiently solvable in case of wireless DAGs with point-to-point links.
- The hardness comes from the requirement of optimally distributing the packets, which is intimately related to Boolean Constraint Satisfaction, described next.

Hardness Reduction: Proof Sketch

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Problem: Does there exist a satisfying assignment such that each clause contain at least one false literal? (Y/N)

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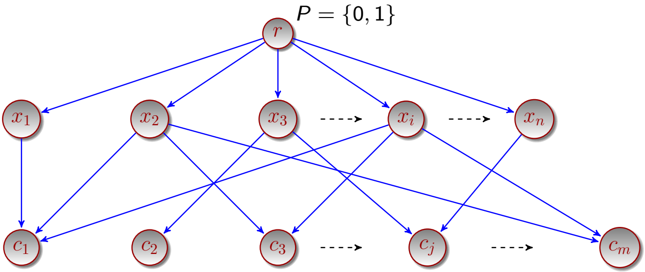

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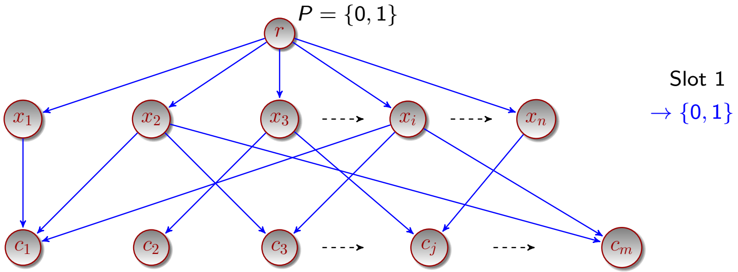

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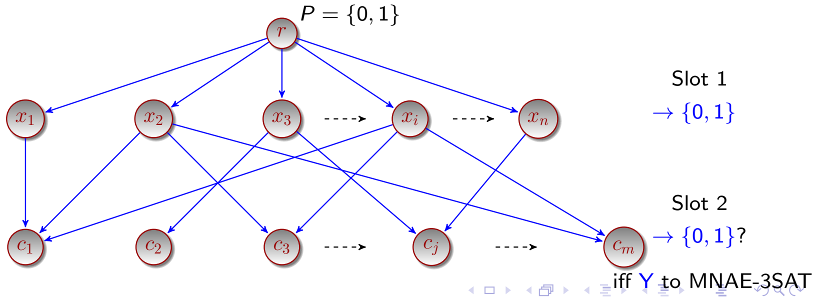

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1. Introduction
2. Hardness of Wireless Broadcasting
3. Throughput-Optimal Broadcast Policy
4. Numerical Results
5. Conclusion

Routing of Packets: Connected Dominating Sets (CDS)

CDS: Defintion

A connected dominating set in a directed graph $\mathcal{G}(V, E)$ and root r is a set of vertices $S \subseteq V$ such that:

- For every $v \in V$, $\exists$ a directed path $\pi = r \to v_1 \to v_2 \ldots v_i \to v$, where all nodes, excepting possibly v, are in the set S.

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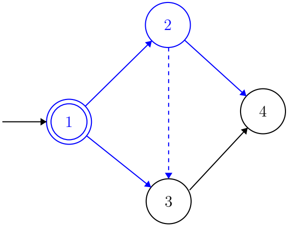

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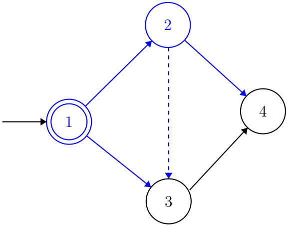

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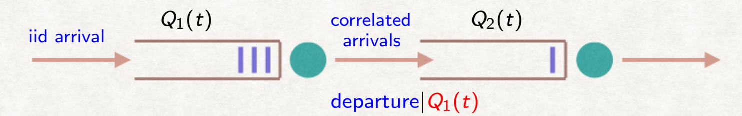
Key observation: Route of a Packet

Every packet must be transmitted sequentially by a CDS in order to be broadcasted.

Design of UMW: Motivation and Insight

- **Observation:** Because of coupling, networked queues are harder to analyze and control.



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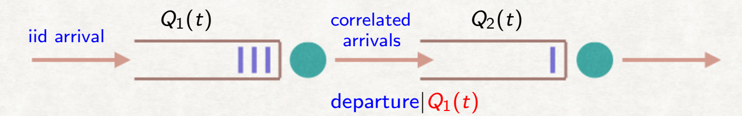

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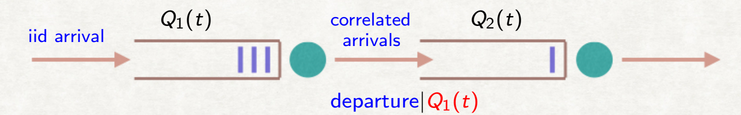

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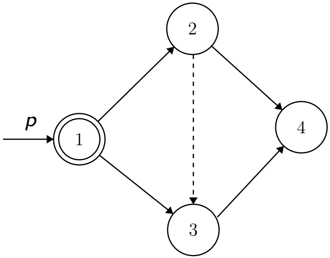
Ans: The Precedence Constraints!

Precedence Constraint: Example

Consider an incoming packet p with the specified broadcasting route $CDS_p = \{1, 2\}$.



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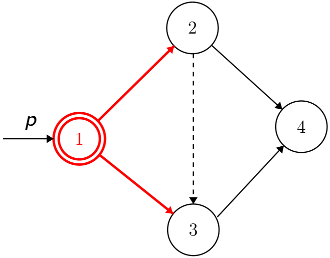

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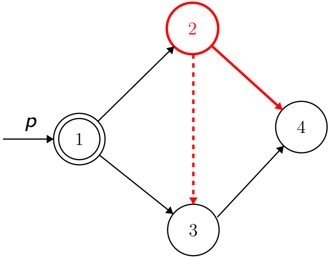
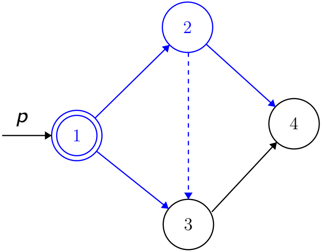


Slot 2

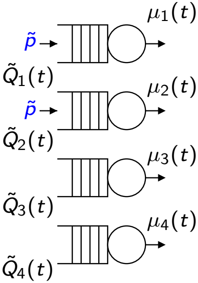
Observation: Due to the precedence, the packet p is transmitted by Node 2 after it has been transmitted by Node 1

Precedence Relaxation: Example

We maintain a virtual system where the packet p is injected to the virtual queues $\tilde{Q}_1, \tilde{Q}_2$ immediately upon arrival.



A Wireless Network $\mathcal{G}$



Virtual Queues

Virtual Queues: Operation

Formally,

1. Associate a virtual queue $\tilde{Q}_v(t)$ with each node v of the graph.
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   - Determine a *CDS* $T_p^*(t)$ for the packet p
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3. Serve the virtual packets at the rate $\boldsymbol{\mu}^*(t)$ as long as the corresponding virtual queues are non-empty
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Question: How to design the optimal controls: $T_p^*(t)$ and $\mu^*(t)$?

Dynamics of the Virtual Queues $\tilde{\boldsymbol{Q}}(t)$

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- Denote the (controlled) arrival to the VQ $\tilde{Q}_i$ by $\tilde{A}_i(t)$. Then, the virtual queues evolve as:

$$\tilde{Q}_i(t+1) = \left(\tilde{Q}_i(t) + \tilde{A}_i(t) - \mu_i(t)\right)^+, \quad \text{(Lindley recursion)} \tag{1}$$

- Note that, the arrivals to the virtual queues $\left(\tilde{A}_i(t), i \in V\right)$ are explicit control variables at the source.
- Unlike the original system, given the controls, the virtual queues are independent of each other. This makes their exact analysis tractable.

Stabilizing Controls for $\tilde{Q}(t)$: Drift Analysis

- A natural first-step is to design $\pi^{\mathtt{UMW}} \equiv \big(\boldsymbol{A}(t), \boldsymbol{\mu}(t)\big)_{t\geq 0}$, such that, it stabilizes the virtual system $\{\tilde{\boldsymbol{Q}}(t)\}_{t\geq 0}$.
- The policy consists of the routing decisions : routing $\boldsymbol{A}^{\pi}(t)$, and scheduling $\boldsymbol{\mu}^{\pi}(t)$.
- Intuition: This control is *likely to stabilize* the physical queues as well
  - However, note that the dynamics of the physical queues depend explicitly on the packet scheduling policy (e.g., FIFO, LIFO etc.). We will come to this issue later.

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  - However, note that the dynamics of the physical queues depend explicitly on the packet scheduling policy (e.g., FIFO, LIFO etc.). We will come to this issue later.
- To stabilize the virtual queues, we choose the control that minimizes the drift of the Quadratic Lyapunov (potential) function of the Virtual Queues.

Derivation of the Control-Policy

- Define a Quadratic Lyapunov (potential) function

$$L(\tilde{\boldsymbol{Q}}(t)) \stackrel{\text{def}}{=} \sum_{i \in V} \tilde{Q}_i^2(t)$$

- The one-slot drift of $L(\tilde{\boldsymbol{Q}}(t))$ under any admissible policy π may be computed to be

$$\begin{aligned} \Delta^{\pi}(t) &\stackrel{\text{def}}{=} L(\tilde{\boldsymbol{Q}}(t+1)) - L(\tilde{\boldsymbol{Q}}(t)) \\ &\leq B + 2\Big(\underbrace{\sum_{i \in V} \tilde{Q}_i(t) A(t) \mathbb{1}(i \in T^{\pi}(t))}_{(a)} - \underbrace{\sum_{i \in V} \tilde{Q}_i(t) \mu_i^{\pi}(t)}_{(b)} \Big) \end{aligned} \quad (2)$$

Where $T^{\pi}(t) \in \mathcal{T}$ and $\boldsymbol{\mu}^{\pi}(t) \in \mathcal{M}$ are routing and activation control variables chosen for slot t.

- The drift upper-bound (2) has a nice separable form and may be minimized over the routing and activation controls individually.

Optimal Routing Policy $T_p^*(t)$

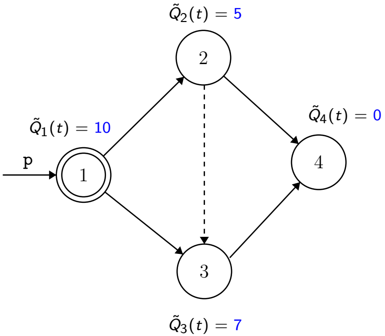
Let $\mathcal{T}$ denote the set of all CDS in $\mathcal{G}$. Minimizing the routing term (a), we get the following optimal routing policy.

Optimal Routing : $\boldsymbol{T_p^*(t)}$

$$T_p^*(t) \in \arg\min_{T \in \mathcal{T}} \sum_{i \in V} \tilde{Q}_i(t) \mathbb{1}(i \in T)$$

In other words, the drift minimizing routing policy is to route the incoming packet along the Minimum Weight CDS, where each node i is weighted by the corresponding virtual queue $\tilde{Q}_i(t)$.

Example of Optimal Routing



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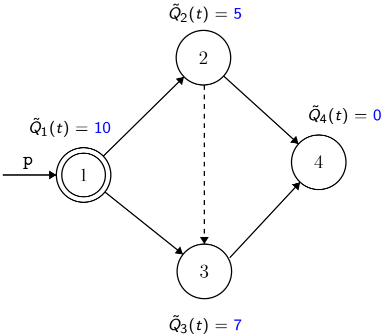

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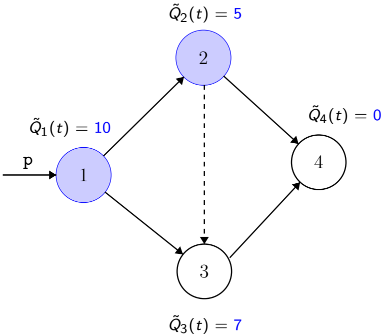

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Chosen route=MCDS= $\{1, 2\}$

Optimal Node Scheduling Policy $\mu^*(t)$

Let $\mathcal{M}$ denote the set of all non-interfering activations in $\mathcal{G}$. Minimizing the scheduling term (b) in the drift expression, we get the following optimal scheduling policy.

Optimal Scheduling : $\boldsymbol{\mu}^*(t)$

$$\mu^*(t) \in \arg\max_{M \in \mathcal{M}} \sum_{i \in V} \tilde{Q}_i(t) \mathbb{1}(i \in M)$$

In other words, the drift minimizing node scheduling policy is to schedule the Max-Weight activation, where each node i is weighted by the corresponding virtual queue length $\tilde{Q}_i(t)$.

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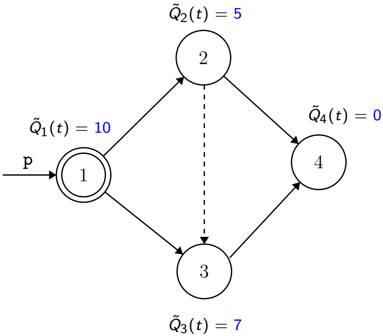

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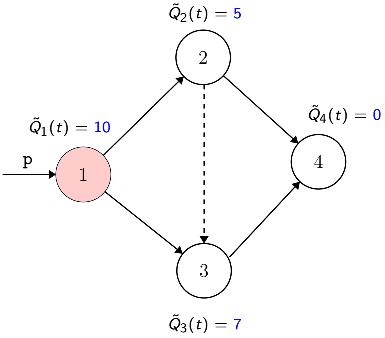

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Optimal Schedule = Activate the Node 1

Stability of the Virtual Queue

Theorem 6: Strong Stability of $\tilde{\boldsymbol{Q}}(t)$

Under the above routing and scheduling policy, for all arrival rate $\boldsymbol{\lambda} \leq \lambda^*$ the virtual queue process is Strongly stable and has a limiting M.G.F, i.e.,

$$\limsup_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T}\sum_{i}\mathbb{E}(\tilde{Q}_i(t)) \leq B$$

and,

$$\limsup_{T\to\infty} \mathbb{E}(\exp(\theta^* \sum_{i} \tilde{Q}_i(t))) \leq C$$

for some finite B, C and strictly positive θ^*.

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The above leads to the following :

Lemma: Sample Path bound on Virtual Queues

Under the same condition, we have

$$\sum_i \tilde{Q}_i(t) = \mathcal{O}(\log t), \quad \text{a.s.}$$

Optimal Control of the Physical Queues : Packet Scheduling

- How do we decide which packet to transmit over a link at any given time slot?
  - Why does it matter? Cannot we just use FCFS?
- Nearest to Origin (NTO) policy [Gamarnik, 1998]
- Extended Nearest to Origin policy (ENTO): When multiple packets contend for an edge, schedule the one which has traversed the least number of edges
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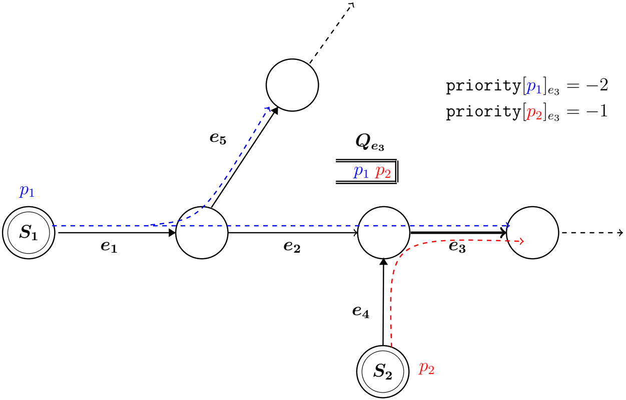
Theorem 7: Stability of the Physical Queues

The overall UMW policy is throughput-optimal.

Proof uses the previous almost sure arrival bound on a typical sample path with an inductive argument on the edges.

ENTO: Example



Packet p_1 has higher priority than p_2 to cross e_3 as it has traversed less number of edges

Proof Ideas for Theorem 7: Stability of the Physical Queues

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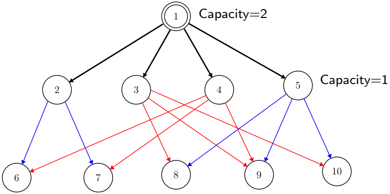
3. With the universal stability property of the ENTO packet scheduling policy it is finally shown that the physical queues are rate stable.

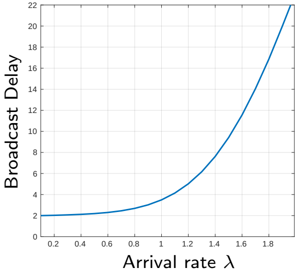
   - Involves induction on the number of hops from the source.

Outline

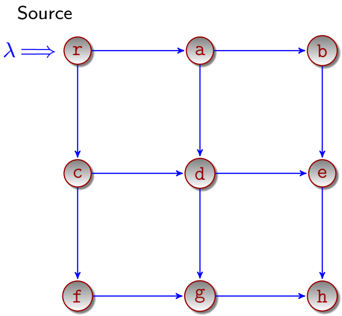
Broadcasting: Network without Interference



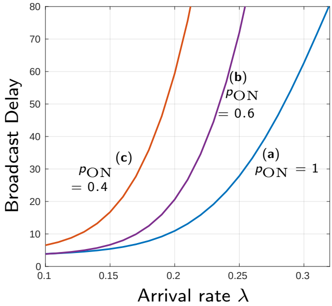
A wireless network with non-interfering channels. The broadcast capacity of the network is $\lambda^* = 2$.



Broadcasting Simulation: Time-Varying Wireless Network



The Grid Network



Plot of the broadcast delay incurred by the UMW policy as a function of the arrival rate λ in the 3×3 wireless grid network.

Outline

Conclusion

- Our understanding of network control theory has progressed enormously over the past 25 years, starting with the seminal Backpressure policy of Tassiulas and Ephremides (1992).
- We have derived a throughput-optimal algorithm, UMW, for broadcasting in wireless networks with **point-to-multipoint** links.
- This important problem was proposed by Massoulie and Twigg, and has remained open for last ten years.
- The virtual network framework used to solve the problem is surprisingly general and may be applied to other open problems in this area (e.g., Sinha, Modiano, INFOCOM '17).
- Opens up exciting new directions for research with lots of interesting problems.