WW scattering

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Outline

1. Introduction

2. $W_L W_L$ scattering and symmetry breaking
   - Equivalence Theorem

3. $W_L W_L$ scattering beyond Higgs mechanism

4. $W_L W_L$ scattering at LHC
   - Equivalent vector-boson approximation
   - Backgrounds
   - Distinguishing the signal

5. Summary
Higgs and WW scattering

- Amplitudes with massive spin-1 particles have worse high-energy behaviour
  \[ \varepsilon_L^\mu(k) \approx \frac{k^\mu}{m_V} \]
- Can make theory non-renormalizable
- In theories with spontaneous symmetry breaking, high-energy behaviour is better
- Higgs exchange can cancel bad high-energy behaviour
- If Higgs mass is too high \((s \ll m_H^2)\), amplitude can be large:
  Strong gauge sector
- Can be studied with WW scattering
$WW$ scattering and symmetry breaking

- The equivalence theorem relates $W_L W_L$ scattering amplitude to the amplitude for scattering of the corresponding “would-be" Goldstone scalars at high energy ($s \gg m^2_W$)
- Goldstone boson interactions are governed by low-energy theorems for energy below the symmetry breaking scale ($s \ll m^2_{SB}$)
- Thus, when there is no light Higgs (with $m_H < 1$ TeV or so), there is a range of $s$ values where the low-energy theorems combined with equivalence theorem can predict $W_L W_L$ scattering amplitudes
- This range is $m^2_W \ll s \ll m^2_{SB}$
- In other words, $W_L W_L$ scattering gives information about the symmetry breaking sector in this range
The Equivalence Theorem

- SM relation

$$m_H^2 = -2\mu^2 = 2\lambda \nu^2 = \lambda \sqrt{2}/G_F$$

- Since $\nu$ and $G_F$ are fixed from experiment, large $m_H$ means large $\lambda$.
- For $m_H \gtrsim (G_F/\sqrt{2})^{-1}$, perturbation theory is not valid.
- This corresponds to $m_H \approx 300$ GeV
- A limit may be obtained from unitarity, if the tree amplitudes are to be valid at high energies.
- Vector boson scattering amplitudes at high energies may be calculated using the equivalence theorem (Dicus & Mathur; Lee, Quigg & Thacker)
Derivation of the equivalence theorem

The scalar Lagrangian is

\[ \mathcal{L} = (D_\mu \phi)^\dagger (D^\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2. \]

We write

\[ \phi = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}. \]

Then \( \phi^\dagger \phi = \frac{1}{2} (\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) \). The Lagrangian thus has a global \( O(4) \) symmetry, under which \( \phi_i \ (i = 1, 2, 3, 4) \) transforms as a four-dimensional representation. Explicitly,

\[ \mathcal{L} = \frac{1}{2} \sum_{i=1}^{4} \left( \partial_\mu \phi_i \partial^\mu \phi_i - \mu^2 \phi_i \phi_i \right) - \frac{\lambda}{4} \left( \sum_{i=1}^{4} \phi_i \phi_i \right)^2, \]

where we have not included the gauge fields, which do not have the full \( SO(4) \equiv SU(2) \times SU(2) \) symmetry.
We define
\[ \vec{\pi} \equiv (\phi_2, \phi_3, \phi_4), \quad \sigma = \phi_1. \]

The Lagrangian is then
\[ \mathcal{L} = \frac{1}{2} \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} \mu^2 \vec{\pi} \cdot \vec{\pi} - \frac{1}{2} \mu^2 \sigma^2 - \frac{\lambda}{4} \left( \vec{\pi} \cdot \vec{\pi} + \sigma^2 \right)^2. \]

The \( O(4) \) is equivalent to \( SU(2)_V \times SU(2)_A \). We now allow for spontaneous symmetry breaking,
\[ \sigma = \phi_1 = \langle \phi_1 \rangle + H = \langle \sigma \rangle + H, \]
with
\[ \langle \sigma \rangle^2 = v^2 = -\mu^2 / \lambda. \]

Rewrite the Lagrangian in terms of the shifted fields:
\[ \mathcal{L} = \frac{1}{2} \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} + \frac{1}{2} \partial_\mu H \partial^\mu H - \frac{1}{2} m^2_H H^2 - \frac{m^2_H}{2v} (H^2 + \vec{\pi}^2) - \frac{m^2_H}{8v^2} (H^2 + \vec{\pi}^2). \]

\( H \) is the only physical field. The \( \vec{\pi} \) fields correspond to the longitudinal degrees of freedom of the massive gauge bosons.
Equivalence theorem (continued)

\[ \pi_3 = z \equiv Z_L \]
\[ \pi^\pm = w^\pm \equiv W^\pm_L. \]

For example, one can check that the \( H \rightarrow W_L W_L \) width comes out right:

\[ M \approx -\frac{m_H^2}{v}, \]
\[ \Gamma = \frac{1}{8\pi} \frac{G_F}{\sqrt{2}} m_H^3. \]
The Higgs cannot be very heavy, or else the unitarity limit would be crossed too soon. To see this, write the old-fashioned scattering amplitude \( f_{cm} \), so that \( |f_{cm}|^2 = \frac{d\sigma}{d\Omega} \). Then we have

\[
f_{cm} = \frac{1}{8\pi\sqrt{s}} \mathcal{M}.
\]

The scattering amplitude has the partial wave expansion

\[
f(\theta) = \frac{1}{k} \sum_l (2l + 1) P_l(\cos \theta) a_l
\]

where \( a_l = e^{i\delta_l} \sin \delta_l \) is the partial wave amplitude written in terms of the phase shift \( \delta_l \). Unitarity is expressed by the optical theorem relation

\[
\sigma = \frac{4\pi}{k} \Im f(0).
\]
Unitarity

For elastic scattering, the phase shift $\delta_l$ is real, but has a positive imaginary part if there is inelasticity. A convenient way to express elastic unitarity is

$$\Im \frac{1}{a_l} = -1.$$

From this it follows that

$$|a_l| \leq 1; \quad |\Re a_l| \leq \frac{1}{2}.$$

The $l = 0$ partial-wave unitarity for large $s$ for $WW$ scattering then gives

$$\frac{G_F m_H^2}{4 \sqrt{2\pi}} < 1,$$

or

$$m_H^2 < \frac{4 \sqrt{2\pi}}{G_F}.$$

This gives a limit of $m_H < 1.2$ TeV.
Models with no light Higgs

- Light Higgs tames high energy behaviour of $W_L W_L$ scattering
- In the absence of light Higgs, $W_L W_L$ interactions become strong
- Violation of unitarity is prevented in different ways, depending on the model
- Models can have extra fermions and extra gauge interactions, which give additional contributions to $W_L W_L$ scattering, e.g., resonances
- An example is techni-rho resonance in techni-color model, or new MVB’s in Higgsless models
- A no-resonance scenario is described in a chiral Lagrangian (EWCL) model, where one can write effective bosonic operators
- Unitarization can be built in by the use of Padé approximants or $N/D$ method and these can generate resonances
**WW contribution vs. others**

Figure: Main diagram topologies for the process

\[ u \rightarrow \overleftrightarrow{d} W \]

(a)

\[ s \rightarrow \overleftrightarrow{c} W \]

Z,γ

(b)

\[ u \rightarrow \overleftrightarrow{d} W \]

Z,γ

\[ s \rightarrow \overleftrightarrow{c} W \]
Equivalent vector-boson approximation

- Equivalent photon approximation (Weizsäcker-Williams approximation) relates cross section for a charged particle beam (virtual photon exchange) to cross section for real photon beam:

\[
\sigma = \int dx \sigma_\gamma(x) f_{q/\gamma}(x)
\]

- Photon distribution with momentum fraction \( x \) in a charged-particle beam of energy \( E \):

\[
f_{e/\gamma}(x) = \frac{q^2 \alpha}{2\pi x} \ln \left( \frac{E}{m_e} \right) [x + (1 - x)^2]
\]

- This is generalized to a process with weak bosons (Dawson; Kane et al.; Lindfors; Godbole & Rindani):

\[
f_{e/V_\pm}(x) = \frac{\alpha}{2\pi x} \ln \left( \frac{E}{m_V} \right) \left[ (v_f \mp a_f)^2 + (1 - x)^2 (v_f \pm a_f)^2 \right]
\]

\[
f_{e/V_L}(x) = \frac{\alpha}{\pi x} (1 - x) \left[ v_f^2 + a_f^2 \right]
\]
Effective vector boson approximation

Use of effective vector boson approximation entails:

- Restricting to vector boson scattering diagrams
- Neglecting diagrams of bremsstrahlung type
- Putting on-shell momenta of the vector bosons which take part in the scattering
- Note that the on-shell point $q_{1,2}^2 = M_{V_{1,2}}^2$ is outside the physical region $q_{1,2}^2 \leq 0$.
- Approximating the total cross section of the process $f_1 f_2 \rightarrow f_3 f_4 V_3 V_4$ by the convolution of the vector boson luminosities $L_{Pol_1 Pol_2}^{V_1 V_2}(x)$ with the on-shell cross section:

$$
\sigma(f_1 f_2 \rightarrow f_3 f_4 V_3 V_4) = \int dx \sum_{V_1, V_2} \sum_{Pol_1 Pol_2} L_{Pol_1 Pol_2}^{V_1 V_2}(x) \times \sigma_{on}^{on}(V_1 V_2 \rightarrow V_3 V_4, xs_{qq})
$$

Here $x = M(V_1 V_2)^2 / s_{qq}$, while $M(V_1 V_2)$ is the vector boson pair invariant mass and $s_{qq}$ is the partonic c.m. energy.
Improved Equivalent vector-boson approximation

- Even if only dominant (?) longitudinal polarization is kept, EVBA overestimates the true cross section.
- Transverse polarization contribution is found to be comparable to longitudinal one (Godbole & Rindani).
- Improved EVBA (Frederick, Olness & Tung), going beyond the leading approximation still overestimates the cross section (Godbole & Olness).
- Further improvements have been attempted (Kuss & Spiesberger).
Backgrounds

- Backgrounds are of two types:
  1. Bremsstrahlung processes – which do not contribute to $VV$ scattering
  2. Processes which fake $VV$ final state

- It is important to understand the first inherent background, and device cuts which may enhance the signal.

- However, it may be possible to live with it – provided $VV$ scattering signal is anyway enhanced because it is strong. In that case, one simply makes predictions for the combined process of $PP \rightarrow VV + X$

- The second background is crucial to take care of, otherwise we do not know if we are seeing a $VV$ pair in the final state or not.
Experimental backgrounds

- Signature of signal is $W_L W_L$ pair in final state
- For no light Higgs TeV region may contain a resonance, or no resonance
- For model with a TeV scalar (spin-0, isospin-0) resonance, most useful detection modes are $W^+ W^-$ and $ZZ$, which contain large isospin-0 channel contributions
- For a model with TeV vector (spin-1, isospin-1) resonance, the most useful mode is $W^\pm Z$ mode
- If no resonances are present, then all $WW$ modes are important, including $W^\pm W^\pm$
- Cleanest final state is through pure leptonic mode of each gauge boson (BR 2/9 for $W$ and 0.06 for $Z$).
Experimental backgrounds

- The intrinsic SM background in the TeV region can be generated by calculating the SM production rate of $ff \rightarrow ffWW$ with a light SM Higgs boson.

- For example, the $W_L W_L$ signal rate in the TeV region from a 1 TeV Higgs boson is equal to the difference between the event rates calculated using a 1 TeV Higgs boson and a 100 GeV Higgs boson.

- Other backgrounds:
  - $W + 2$ jets (“fake $W$" mimicked by two QCD jets)
  - $\bar{t}t$ pair (which subsequently decays into a $WW$ pair)
Backgrounds in $ZZ$ leptonic decay modes

- Signal is an event with four isolated leptons with high $p_T$ (for both $Z$ decaying into $\ell^+\ell^-$) or two isolated leptons associated with large missing $p_T$ (for one $Z$ decaying into $\ell^+\ell^-$ and the other to $\nu\bar{\nu}$).
- Dominant background processes are $q\bar{q} \rightarrow ZZX$, $gg \rightarrow ZZX$ where the $X$ can be additional QCD jet(s).
- Final state $Z$ pairs produced in these processes tend to be transversely polarized.
- Also, $Z$ pairs produced from intrinsic electroweak background process are mostly transversely polarized.
- This is because coupling of a transverse $W$ to a light fermion is stronger than that of a longitudinal $W$ at high-energy.
- Kinematic cuts needed to enhance events with longitudinally polarized $W$ emitted from incoming fermions, and hence enhance $W_L W_L \rightarrow W_L W_L$ signal event.
Background in leptonic decay of $W^+ W^-$

For $W^+ (\rightarrow \ell^+ \nu) W^- (\rightarrow \ell^- \bar{\nu})$ mode

- Background processes $q\bar{q} \rightarrow W^+ W^- X$, $gg \rightarrow W^+ W^- X$
- $t\bar{t} + \text{jet}$, with top decays giving $W^+ W^-$ pair
Background in semi-leptonic decay of $W^+W^-$

For $W^+(\rightarrow \ell^+ \nu)W^-(\rightarrow q_1 \bar{q}_2)$ mode

- The signature is isolated lepton with high $p_T$, large missing $p_T$, and two jets with invariant mass about $m_W$.

- Electroweak-QCD process $W^+ +$ jets can mimic the signal when the invariant mass of the two jets is around $m_W$.

- Potential background from QCD processes $q\bar{q}, gg \rightarrow t\bar{t}X, Wt\bar{b}$ and $t\bar{t} +$jets), in which a $W$ can come from the decay of $t$ or $\bar{t}$. 

Distinguishing signal from background

We concentrate on $W^+ (\rightarrow \ell^+ \nu) W^- (\rightarrow q_1 \bar{q}_2)$ mode

- $W$ boson pairs produced from the intrinsic electroweak process $q\bar{q} \rightarrow q\bar{q} W^+ W^-$ tend to be transversely polarized
- Coupling to $W^+$ of incoming quark is purely left-handed
- Hence helicity conservation implies that outgoing quark follows the direction of incoming quark for longitudinal $W$, and it goes opposite to direction of incoming quark for transverse (left-handed) $W$
- Hence outgoing quark jet is less forward in background than in signal event, and tagging of the forward jet can help
- Charged particle multiplicity of the signal event (purely electroweak) is smaller than that typical electroweak-QCD process like $q\bar{q} \rightarrow gW^+ W^-$
- One can reject events with more than two hard jets in central rapidity region, which would also remove background from $t\bar{t}$ production
Other ways of reducing background

1. Isolated lepton in $W^+ \rightarrow \ell^+ \nu$
   - Background event has more hadronic activity in the central rapidity region
   - Hence the lepton produced from $W$ decay in background event is less isolated than that in signal
   -Requiring isolated lepton with high $p_T$ is thus useful for suppressing background

2. $W \rightarrow q_1 \bar{q}_2$ decay mode
   - After imposing veto of central jet and tagging of forward jet, $t\bar{t}$ background is small
   - Can be further suppressed by vetoing event with $b$ jet coming from top decay
Axial gauge vs. unitary gauge vs. EVBA vs. exact results

- The feasibility of extracting $WW$ scattering from experiment and comparison of EVBA with exact results was recently studied by Accomando et al., hep-ph/0608019.
- It is known that when $W$’s are allowed to be off mass shell, amplitude grows faster with energy, as compared to when they are on shell (Kleiss & Stirling, 1986).
- Problem of bad high-energy behavious of $WW$ scattering diagrams can be avoided by the use of axial gauge (Kunszt and Soper 1988).
- In axial gauge, Goldstone and gauge fields mix, with the gauge propagator given by

$$
\left( -g_{\mu\nu} + \frac{q_{\mu} n_{\nu} + n_{\mu} q_{\nu}}{q \cdot n} - \frac{n^2}{(q \cdot n)^2} q_{\mu} q_{\nu} \right) (q^2 - m_W^2)^{-1}
$$
Axial gauge vs. unitary gauge vs. EVBA vs. exact results

Accomando et al. examine
- Role of choice of gauge in $WW$ fusion
- Reliability of EVBA
- Determination of regions of phase space, in suitable gauge, which are dominated by the signal ($WW$ scattering diagrams)

Results show that
- $WW$ scattering diagrams do not constitute the dominant contribution in any gauge or phase space region
- There is no substitute to the complete amplitude for studying $WW$ fusion process at LHC
### $WW$ contribution vs. others

#### No Higgs

<table>
<thead>
<tr>
<th>Gauge</th>
<th>$\sigma (pb)$</th>
<th>$WW$ diagrams</th>
<th>ratio $WW/all$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unitary</td>
<td>$1.86 \times 10^{-2}$</td>
<td>6.67</td>
<td>358</td>
</tr>
<tr>
<td>Feynman</td>
<td>$1.86 \times 10^{-2}$</td>
<td>0.245</td>
<td>13</td>
</tr>
<tr>
<td>Axial</td>
<td>$1.86 \times 10^{-2}$</td>
<td>$3.71 \times 10^{-2}$</td>
<td>2</td>
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</tbody>
</table>

**Table:** No Higgs contribution, using the CTEQ5 Pdf set with scale $M_W$

$M_H = 200$ GeV

<table>
<thead>
<tr>
<th>Gauge</th>
<th>$\sigma (pb)$</th>
<th>$WW$ diagrams</th>
<th>ratio $WW/all$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unitary</td>
<td>$8.50 \times 10^{-3}$</td>
<td>6.5</td>
<td>765</td>
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<tr>
<td>Feynman</td>
<td>$8.50 \times 10^{-3}$</td>
<td>0.221</td>
<td>26</td>
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<tr>
<td>Axial</td>
<td>$8.50 \times 10^{-3}$</td>
<td>$2.0 \times 10^{-2}$</td>
<td>2.3</td>
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</table>

**Table:** $M_h = 200$ GeV Higgs and $M(WW) > 300$ GeV
**WW invariant mass distribution**

**Figure:** Distribution of \( d\sigma/dM_{WW} \) for the process \( PP \to us \to cdW^+ W^- \) for All diagrams, \( WW \) diagrams and their ratio in Unitary, Feynman and Axial gauge in the infinite Higgs mass limit. The Unitary gauge data in the left hand plot have been divided by 20 for better presentation.
Comparison with EVBA results: \( WW \) invariant mass

**Figure:** \( WW \) invariant mass distribution \( M( WW ) \) for the process \( us \rightarrow dcW^+ W^- \) with EVBA (black solid curve) and with exact complete computation (red dashed curve) for no Higgs (left) and \( M_h = 250 \) GeV (right)
Comparison with EVBA: Total cross section

<table>
<thead>
<tr>
<th>$M_h$</th>
<th>EVBA (pb)</th>
<th>EXACT (pb)</th>
<th>Ratio</th>
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</thead>
<tbody>
<tr>
<td>$\infty$</td>
<td>$3.90 \times 10^{-2}$</td>
<td>$1.78 \times 10^{-2}$</td>
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<td>130 GeV</td>
<td>$3.94 \times 10^{-2}$</td>
<td>$1.71 \times 10^{-2}$</td>
<td>2.3</td>
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<tr>
<td>250 GeV</td>
<td>$4.61 \times 10^{-2}$</td>
<td>$4.09 \times 10^{-2}$</td>
<td>1.12</td>
</tr>
<tr>
<td>500 GeV</td>
<td>$4.42 \times 10^{-2}$</td>
<td>$2.5 \times 10^{-2}$</td>
<td>1.77</td>
</tr>
</tbody>
</table>

**Table:** Total cross sections computed with EVBA and exact computation and their ratio for the process $us \rightarrow cdW^+ W^-$ at fixed CM energy $\sqrt{s} = 1$ TeV.

<table>
<thead>
<tr>
<th>$\theta_{cut}$</th>
<th>EVBA (pb)</th>
<th>EXACT (pb)</th>
<th>Ratio</th>
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<tbody>
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<td>$10^\circ$</td>
<td>$4.42 \times 10^{-2}$</td>
<td>$2.5 \times 10^{-2}$</td>
<td>1.77</td>
</tr>
<tr>
<td>$30^\circ$</td>
<td>$1.33 \times 10^{-2}$</td>
<td>$2.06 \times 10^{-2}$</td>
<td>0.64</td>
</tr>
<tr>
<td>$60^\circ$</td>
<td>$6.06 \times 10^{-3}$</td>
<td>$1.28 \times 10^{-2}$</td>
<td>0.47</td>
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</table>

**Table:** Total cross section in EVBA and exact computation and their ratio for different angular cuts. The CM energy is $\sqrt{s} = 1$ TeV and the Higgs mass $M_h = 500$ GeV.
Large invariant mass region

The $WW$ invariant mass distribution in $PP \rightarrow us \rightarrow cdW^+W^-$ for no Higgs mass (solid curves) and for $M_h=200$ GeV (dashed curve). The two intermediate (red) curves are obtained imposing cuts shown below.

The two lowest (blue) curves refer to the process $PP \rightarrow us \rightarrow cd\mu^-\bar{\nu}_\mu e^+\nu_e$ with further acceptance cuts: $E_l > 20$ GeV, $p_T > 10$ GeV, $|\eta| < 3$.

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<table>
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<tbody>
<tr>
<td>$E$(quarks)$&gt; 20$ GeV</td>
<td></td>
</tr>
<tr>
<td>$P_T$(quarks,W)$&gt; 10$ GeV</td>
<td>$2 &lt;</td>
</tr>
<tr>
<td></td>
<td>$</td>
</tr>
</tbody>
</table>

**Table:** Selection cuts applied
Some references

- **WW scattering in the context of heavy Higgs boson:**


- **“Off-shell scattering amplitudes for WW scattering and the role of the photon pole”,** J. Bartels and F. Schwennen, PRD 72, 034035 (2005)
Summary

- In the absence of a light Higgs, $WW$ interactions become strong at TeV scales.
- Study of $WW$ scattering can give information of the electroweak symmetry breaking sector and discriminate between models.
- In general there are large cancellations between the scattering and bremsstrahlung diagrams.
- Hence extraction of $WW$ scattering contribution from the process $PP \rightarrow W^+ W^- X$ needs considerable effort.
- EVBA overestimates the magnitude in most kinematic distributions.
- Cuts to reduce background were discussed.