Dynamics of a Cheetah's Run

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Abstract

A cheetah belongs to the family of cats who are amongst the swiftest in their movements and whose speed can match modern technology. However, such swift motion implies large consumption of energy and places several physiological limitations. This article deals with the physics, chemistry and biology of a cheetah, which is in the spirit of the Junior Science Olympiad to integrate the sciences and to unify concepts. The article brings to the fore many facts about the Cheetah and, equally importantly, shows how many of the Cheetah's attributes can be calculated by students with a minimal set of assumptions. Some of the features of the Cheetah look unrealistic, but are true, revealing the extreme manifestations of Nature when it comes to the Cheetah.

Cheetah, known as *Acinonyx jubatus,* is the fastest running land animal. Now extinct from India, it is found mostly in Africa and Iran. It can accelerate from rest to its maximum running speed of about 30 m s^{-1} (about 100 km/hour) in just 3.0 s, which amounts to a most impressive acceleration. By comparison, consider that a fast sports car - a Porsche - takes about 4.0 s to attain the same speed ["Cheetah: fast facts", Zoological Society of London. Retrieved 5 June 2013.] Its speed is the Cheetah's main weapon in hunting where it can swoop down on preys very swiftly and at the same time, it is also its defense from predators. But this maximum speed requires large effort by the Cheetah where it has to move its body machinery with unusually high efficiency, consuming lots of energy and, also, generating a lot of fatigue. Thus, this kind of high speed is not sustainable for a very long time and the Cheetah can run at this speed for distances longer than 500 m. The longest unsuccessful chase at very high speeds has been recorded to last a distance covering 650m.

The head-to-tail (base) length of a typical Cheetah ranges from 110 cm to 150 cm (43 to 59 in). The length of the tail can measure from 60 to 84 cm (Ref: Wikipedia). At the front legs, Cheetahs are around 66 to 94 cm tall. Compared to a leopard, Cheetah have a shorter body, but are longer, taller and have longer tails. At high speeds, a Cheetah breathes about 60 to 150 times a minute. A Cheetah weighs between 45 to 60 kg.

A Cheetah reaches its maximum velocity in around 3 seconds and can run at its maximum speed for around 20 or 21 seconds before it needs to slow down. In this duration its body temperature rises by around 1.5 degrees. Any further rise in body temperature can turn fatal for a Cheetah. In this duration a Cheetah requires more than 50 litres of oxygen but can often manage with a smaller quantity. A Cheetah can identify its prey at a distance of around 5 km.

When running at high speeds, a Cheetah takes only about 220 milliseconds to swing its legs back to the front after having finished a stride. In each stride a Cheetah jumps around 7 m. This is the limiting factor on how fast it can run.

For a student it is very interesting to understand the dynamics of a Cheetah as it runs to capture its prey. Students can understand how a Cheetah gets its energy from different manifestations of respiration. The example of a Cheetah also highlights how Nature functions at the extreme and how speed, a weapon for the Cheetah, is also its drawback and limiting factor. The questions formed below have several assumptions by which a student of secondary school level can calculate the numbers with ease.

Image taken from: http://www.vimeo.com

As already noted, although a Cheetah can accelerate and run very fast, it cannot run a long distance at its maximum speed because it quickly gets tired. Thus, if it cannot catch its prey within that limit, it has to forgo the hunt. This article discusses the dynamics of a Cheetah's run, some of the physics that governs its running, energy production and energy consumption, and the biological processes that are involved. The solutions to the questions are given in red.

1) Consider a Cheetah of mass 50 kg. It starts from rest and accelerates for 3.0 s to reach its maximum speed of 30 m s^{-1}. It then continues to run for 20 s at this speed.

(i) Calculate the average acceleration required to reach its maximum speed.

[0.5]

(ii) Calculate the distance travelled during the first 3.0 s, assuming that the acceleration is uniform.

[0.5]

(iii) The Cheetah has to do work against friction, mostly due to air. Assume that this frictional force is always 100 N. Calculate the total mechanical work done by the Cheetah during the first 23 s of its motion.

[1.0]

2) During the first 23 s, the body temperature of the Cheetah rises from 38.5 ^{0}C to 40.0 ^{0}C. Take the specific heat of the body of the Cheetah to be 4.2 kJ kg^{-1} K^{-1}.

(i) If the rise in body temperature is linear during this time, calculate the total heat generated by the Cheetah's metabolism. Neglect any heat loss to the surroundings.

[1.0]

(ii) Assume that some of the energy generated by the Cheetah's body increases its temperature and the rest corresponds to the mechanical work done. Calculate the fraction of the total generated energy that is converted to kinetic energy.

[**1.0]**

3) When the Cheetah starts running, it generates its energy initially by aerobic respiration, where glucose is oxidised in the presence of oxygen, resulting in generation of ATP. In this process, each mole of glucose generates 36 moles of ATP, and 1130 kJ energy is released when all these ATP molecules are utilised. Running at high speeds increases the demand for oxygen, resulting in the increased breathing rate of 150 breaths per minute.

(i) Write down the balanced chemical reaction for aerobic respiration.

[1.0]

(ii) If the Cheetah requires 400 kJ of energy, calculate the volume of oxygen required if all this energy is to be obtained by aerobic respiration. Take the molar volume of oxygen gas to be 24.5 litres.

[1.0]

(iii) The Cheetah extracts oxygen from the air while breathing. The inhaled air (about 500 ml per breath) contains 20.0 % oxygen (by volume), while the exhaled air is assumed to contain 15.0 % oxygen (by volume). Calculate the volume of oxygen that the cheetah can use during the 23.0 s of its run, at a breathing rate of 150 breaths per minute.

[1.0]

4) It should be clear from the answers to the above that the energy requirement of the Cheetah's muscles is not met only by aerobic respiration. ATP must then be produced by anaerobic respiration, but in this only two moles of ATP are generated per mole of glucose.

(i) Anaerobic respiration converts the energy from glucose into ATP. If glucose were to be completely burnt up, one mole would release 2872 kJ of energy. What is the efficiency of anaerobic respiration compared to complete combustion of glucose?

[1.0]

(ii) If all the 400 kJ required by the Cheetah for its run were to be produced by anaerobic respiration, calculate the total amount of glucose (in kg) that would be required.

[1.5]

Solutions to the questions.

1) i) Calculate the average acceleration of this cheetah required to reach its maximum speed.

[0.5]

Average acceleration, $a = \frac{v}{t}$

$a = \frac{30}{3} = 10ms^{-2}$

ii) Calculate the distance travelled during the first 3.0 s, assuming that the acceleration is uniform.

[0.5]

Distance travelled during the acceleration phase , $d^1 = \frac{1}{2}at^2$

$d^1 = \frac{1}{2} \times 10 \times 3^2 = 45m$

iii) The Cheetah has to do work against friction, mostly due to air. Assume that this frictional force is always 100 N. Calculate the total mechanical work done by the cheetah during the first 23.0 s of its motion.

[1.0]

Work done in

Change in kinetic energy, $K = \frac{1}{2}mv^2 - 0$

$K = \frac{1}{2} \times 50 \times 30^2 = 22.5kJ$

Overcoming friction due to air, $W_d = F_a \times (d + d^1)$

$W_d = 100 \times (600 + 45) = 64.5kJ$

Total work done $W = K + W_d$

$W = 22.5 + 64.5 = 87kJ$

2) During the first 23.0 s, the body temperature of the cheetah rises from 38.5 ^{0}C to 40.0 ^{0}C. Take the specific heat of the body of the Cheetah to be 4.2 kJ kg^{-1} K^{-1}.

i) If the rise in body temperature is linear during this time, calculate the total heat generated by the Cheetah's metabolism. Neglect any heat loss to the surroundings. [Here it is assumed that cheetah is completely made of water]

[1.0]

Heat generated in the body of the Cheetah, $H = Sm\Delta t$

$$H = 4.2 \times 50 \times (40 - 38.5) = 315kJ$$

ii) Assume that some of the energy generated by the Cheetah's body increases its temperature and the rest corresponds to the mechanical work done. Calculate the fraction of the total generated energy that is converted to kinetic energy.

[**1.0]**

Total energy generated in the body of Cheetah, $E = H + W$

$E = 315 + 87 = 402kJ$ Fraction $= \frac{22.5}{402} = 0.06$

3) i) Write down the balanced chemical reaction for aerobic respiration.

[1.0]

The balanced reaction: $C_6H_{12}O_6 + 6O_2 \longrightarrow 6CO_2 + 6H_2O$

ii) If the cheetah requires 400 kJ of energy, calculate the volume of oxygen required if all this energy is to be obtained by aerobic respiration. Take the molar volume of oxygen gas to be 24.5 litres.

[1.0]

Number of glucose molecules required to produce 400kJ of energy is

$$n_g = \frac{400\ kJ}{U}$$

$$n_g = \frac{400\ kJ}{U1130\ kJ} = 0.35\ mol$$

Number of mol of O_2 is $n_O = 6n_g$

$n_O = 6 \times 0.35 = 2.1\ mol$

Volume of O_2 required, $V = 24.5\ \times n_O = 24.5 \times 2.1 = 52\ l$

iii) The Cheetah extracts oxygen from the air while breathing. The inhaled air (about 500 ml per breath) contains 20.0 % oxygen (by volume), while the exhaled air is assumed to contain 15.0 % oxygen (by volume). Calculate the volume of oxygen that the cheetah can use during the 23.0 s of its run, at a breathing rate of 150 breaths per minute.

[1.0]

Amount of O_2 absorbed by the lungs per breath is $V^1 = 5\%\ of\ 500ml = 25ml$

Total number of breaths during its entire motion,

$$n = \frac{N}{60} \times total\ duration\ of\ motion$$

$$n = \frac{150}{60} \times 23 = 57.5 \approx 58$$

Thus the amount of intake of O_2, $V_a = n \times V^1 = 58 \times 25 = 1.45l$

4) i) Anaerobic respiration converts the energy from glucose into ATP. If glucose were to be completely burnt up, one mole would release (U^1) 2872 kJ of energy. What is the efficiency of anaerobic respiration compared to complete combustion of glucose?

[1.0]

Energy generated in production of one mol of ATP due to oxidation of glucose is

$$U_g = \frac{u}{36} = \frac{113.0}{36} = 31.4kJ$$

Efficiency of the anaerobic respiration, $\eta = \frac{U_g}{U^1}$

$$\eta = \frac{2 \times 31.4}{2872} = 0.022$$

ii) If all the 400 kJ required by the cheetah for its run were to be produced by anaerobic respiration, calculate the total amount of glucose (in kg) that would be required.

[1.5]

Amount of glucose of required, $N = \frac{400}{2U_g}$

$N = \frac{400}{62.8} = 6.4\ mol$

Approximate molecular mass of glucose,

$M = 6 \times 12 + 12 \times 1 + 6 \times 16 = 180g$

$m = 180 \times 6.4 = 1.2\ kg$

The article reveals some of the interesting and unbelievable but true features of Cheetahs. The acceleration of a Cheetah is almost same as that of a body falling under the influence of gravity on the surface of earth. At such high value of acceleration and the velocity at which it runs, of the total energy consumed, only 6 percent goes in changing the Cheetah's kinetic energy. If it were to breathe normally, in a duration of 23 seconds a Cheetah would need 50 *l* of Oxygen which is a huge amount considering the fact that Cheetah weighs of the order of 40 to 60 Kg. However, given the respiratory structure of a Cheetah, it cannot breath more than 1.5 *l* of oxygen in this duration. Thus switching over to anerobic respiration it consumes nearly 1 kg of glucose to achieve this feat in about 20-25 seconds of running. After calculating these numbers, a student can well imagine the extreme conditions that a Cheetah can routinely set itself up against, for short durations, in order to achieve its kill.

The calculations on the basis of several simplifications give student an idea of how a Cheetah consumes food (quantity in terms of glucose), converts it into the energy required to run at high accelerations and high speeds. This question was part of the International Junior Science Olympiad 2013, held at Pune. The performance of 228 students is presented in the chart below. This question was an interdisciplinary question dealing with the Biology, Chemistry and Physics of a Cheetah.