Measurement of Moment of Inertia

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Moment of Inertia (I) is the "mass property of a rigid body that determines the torque needed for a desired angular acceleration about an axis of rotation". It can also be defined as a measure of rotational inertia. Generally, text books give details of how to calculate I for different bodies about different axes of rotation. Experiments are also prescribed for bodies having regular shapes. But very rarely do textbooks discuss about how to experimentally measure I for bodies having irregular shapes. The following experiment is designed to measure the value of I for a two dimensional triangular piece of cardboard.

The triangular piece is considered as a rigid body where every particle of the body is at a fixed distance from every other *particle of the* body. *As it is a rigid body, this distance does not change* under application of an external force. This experiment is carried out along similar lines to the experiment "physical pendulum".

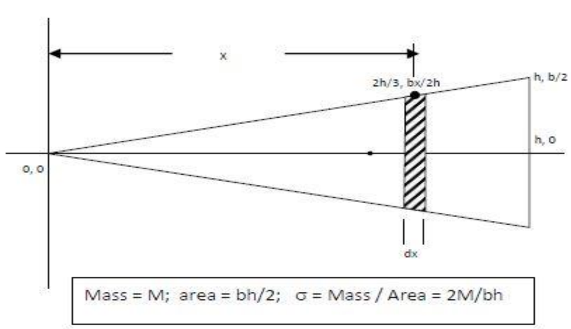
When a rigid body rotates, every particle describes a circular path. The centres of all these circles fall on the same straight line which is known as the axis of rotation. The moment of Inertia *I*, of a rigid body about one particular axis of rotation can be deduced using the equation

$$I = \sum_i m_i r_i^2 \qquad (1)$$

where m_i is the mass of the i^{th} particle at a distance r_i from the axis of rotation. Thus the Moment of Inertia of a body will depend on the mass of the body and its distribution about the axis of rotation.

The theoretical equation to calculate moment of inertia *about an axis of rotation passing through center of mass and perpendicular to the plane of the triangle* is

$$I_c = (M/72)[\,4h^2 + 3b^2] \qquad (2)$$



We know that a *simple pendulum* consists of a point mass m suspended from a string of fixed length l and negligible mass, the other end of which is fixed to a rigid support O. For small displacements from the equilibrium position (shown in the figure below), the point mass m executes simple harmonic motion with time period T (time taken for one oscillation):

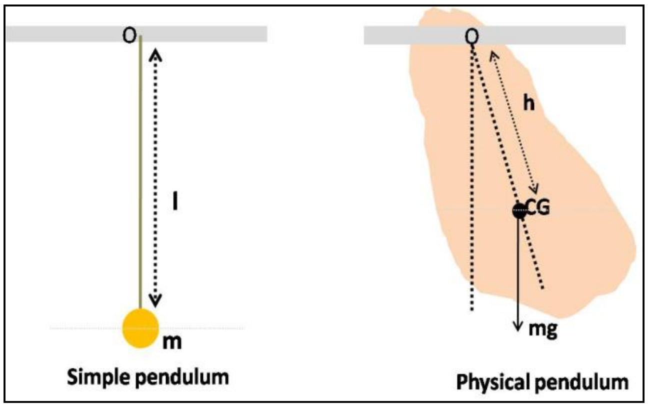
$$T = 2\pi\sqrt{\frac{l}{g}} \qquad (3)$$

where g is the acceleration due to gravity.

A much wider variety of situations that involve small oscillations may be described in terms of a *physical pendulum*, also called a "compound pendulum". Using this concept, we can describe the motion of a rigid body of mass m, of arbitrary shape and size. It is pivoted at O (called the "point of suspension", as shown in the above figure). For small displacements, such a physical pendulum executes simple harmonic motion, with time period

$$T = 2\pi\sqrt{\frac{I_0}{mgh}} \tag{4}$$

Here I_0 is the *moment of inertia* about an axis passing through the point of suspension, h is the distance of the point of suspension from the *centre of gravity* (CG), and g is the acceleration due to gravity.



In this experiment we consider a triangular plate of mass m which oscillates in its own plane. Its moment of inertia about an axis passing through its point of suspension O is given by:
$I_0 = m\,(K^2 + h^2)$
where K is called the radius of gyration.
Theoretically this is given by
$I_o = (M/24)[\ 12h^2 + b^2]$ and $I_o = I_c + Mh^2$

The time period of oscillation of the physical pendulum is therefore

$$T = 2\pi\sqrt{\frac{K^2+h^2}{gh}} \tag{5}$$

The time period can also be written as $T = 2\pi\sqrt{\frac{L}{g}}$
where $L = \frac{K^2}{h} + h$ is called the length of an equivalent simple pendulum.

A point S, on the other side of the CG and at a distance of $h' = \frac{K^2}{h}$ from the CG (along the line joining O and CG) is called the "point of oscillation". The oscillations with the point of suspension O are then equivalent to having all the mass concentrated at S.

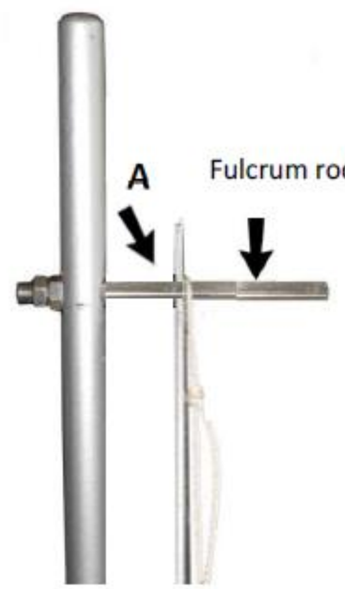
You are supplied with the following:

	Quantity
Clamp Stand	1
Triangular plate (acrylic or cardboard)	1
Fulcrum rod with knife edge for suspension	1
Plumb line	1
Ruler	1
Stop watch Same stopwatch to be used for Task B	1

To determine the centre of gravity (CG) of the triangular plate A.

Procedure:

1. Suspend the triangular plate **A** from the fulcrum rod (mounted on the clamp stand) by one of the holes provided, closest to one of the corners. (see the figure below).



2. Ensure that the suspended plate is stationary. Pass the loop of the string on the rod and hang the plumb line through the fulcrum rod (as shown in the figure above). Using a ruler and pencil, mark a straight line on the plate along the string.

3. Repeat the same procedure by suspending the plate through a different hole, again nearest to the corner. The intersection of the two lines gives the CG. Use a pencil to mark it as 'X' on the plate.

   Mark the two lines and the point 'X' also on the sheet of paper (provided to you) with a drawing of the triangular plate on it. Label it as **Sheet 1**. [X marks]

4. Suspend the plate through a different hole, near the corner and repeat steps 1 and 2. This line should also pass through the CG. Show the line on **Sheet 1** also.

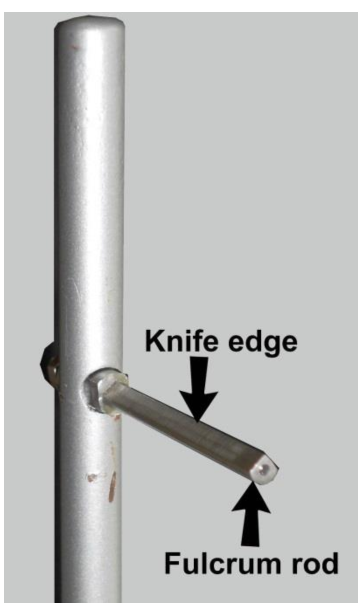
Note: The correct determination of CG is very important as any error here will introduce a corresponding error in the measurement of h, which will be used later.

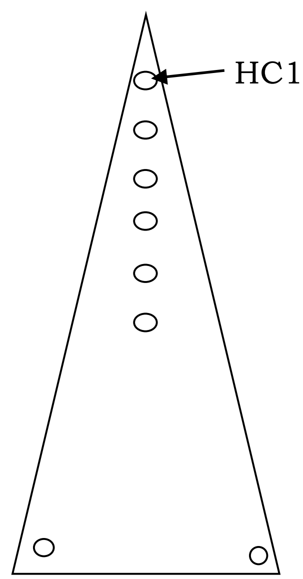
[Note: this is a part which requires skill and observation of the students. Students tend to draw the line while the piece is still suspended from the fulcrum point. It is possible that the while drawing the line they can disturb the triangle and hence get a wrong CG point. It is best to just mark the points on paper and then draw straight lines to determine the CG. The teacher is the best judge to test skills of students]

<u>To record the time period of oscillation for different suspension points for the plate.</u>

Procedure:

1. Suspend the plate from hole **HC1** using the fulcrum rod. Ensure that the plate is almost at the centre of the fulcrum rod and resting on the knife edge (see the figure below). This is important to reduce damping of the oscillations and, hence, to minimize error in determining the time period of oscillations.





Note: Measure all distances from the top end of all the holes.

2. Measure the distance **h** between the hole **HC1** and the **CG** you marked in the previous part of the experiment. (Measure the distance from the top end of the hole **HC1**). Write it in **Table A.1**

3. Set the plate into oscillation (with small amplitude) and ensure that these oscillations occur mostly in the plane of the plate.

4. Using the stop watch, measure the time taken for 50 oscillations. Repeat three times and write each reading in **Table A.1.**

5. Repeat the above steps for holes **HC2**, **HC3**, **HC4** and **HC5.** **[X marks]**

A3 To analyze the above data and determine

a) the acceleration due to gravity
b) the radius of gyration of the plate about an axis passing through its CG normal to the plane of the triangle;
c) the positions of the corresponding points of oscillation from the CG for two points of suspension; and
d) the lengths of the equivalent simple pendulum for these two points of suspension.

Procedure for *Data Analysis*:

1. Using the data in **Table A.1**, plot a graph of hT^2(y-axis in ms^2) versus h^2 (x-axis in m^2) on the grid provided in the answer sheet (**Grid 1**). **[X marks]**

2. Draw a straight line through the points (best fit) and determine the slope s and the y-intercept c.

   Using these values of s and c, and the expression for the time period of a physical pendulum, determine the values of g in ms^{-2} and K in units of meters. Enter the values of s, c, g, and K in **Table A.2.** **[X marks]**

3. For holes **H1** and **H5**, calculate the positions of the corresponding points of oscillation from the CG (h'). Write it in **Table A.3**. On the sheet of paper (**Sheet 1**), mark the positions of the points of oscillation **J1** and **J5** corresponding to the holes **H1** and **H5, respectively.** **[X marks]**

By parallel axes theorem

$$I = I_c + Mh^2 \tag{6}$$

1. From the values of M & I_c from equation (2) and values of h recorded in table 1 calculate the values of the value of I about different holes using the parallel axes theorem. Note down these values in the column for I_T in table 2.
2. Calculate the values of K from slope of intercept and k_i using equation (5) and (6) and enter in Table 2 in the answer sheet.
3. With these values of k_i, calculate I_E in each case using equation (1) and enter the values in table 1.
4. Determine the % deviation in each case.

Answer sheet

A.Q1 Determination of CG: **[X mark]**

Mark "X" on **Sheet 1** at the appropriate position to denote the CG (large sized sheet).

A.Q2 Table A.1: Oscillation measurements: **[X marks]**

	h (m)	**h^2 (m^2)**	**Time taken for 50 oscillations (s)**				**T = T1/50 (s)**	**T^2 (s^2)**	**hT^2 (ms^2)**
			1st (t1)	**2nd (t2)**	**3rd (t3)**	**Mean (T1) (t1+t2+t3)/3**			
H1									
H2									
H3									
H4									
H5									

A.Q3 Results of the data analysis

(a) Grid 1: hT^2 (y-axis) versus h^2 (x-axis) **[X marks]**

A.Q4 Table A.2: Calculations from Grid 1 **[X marks]**

Quantity	**Numerical value**	**Unit**
Slope of the graph (s)		
y-intercept of the graph (c)		
Acceleration due to gravity (g)		
Radius of gyration (K)		

A.Q5 (a) Table A.3: **[X marks]**

Holes	**h (m)**	**h' (m)**
H1		
H4		

(b) Sheet 1: Mark the positions of points of oscillation J1 and J4 on **Sheet 1**. Label them as J1 and J4 clearly.

A.Q6 Table A.4: Lengths of equivalent simple pendulums **[X mark]**

Holes	**h (m)**	**L (m)**
H1		
H4		

Graph plotting (X marks)

g = ______________________ (X marks)

k=__________________________ (X marks)

Table 2

Hole number	I_T /g-cm^2	h/cm	T^2/s^2	k_i /cm	I_E/ g-cm^2	% deviation = (I_E - I_T)*100/I_T
1						
2						
3						
4						
5						
6						