# Physical pendulum

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Abstract

A simple pendulum experiment is carried out widely in schools and students are familiar with theory of pendulum. However, if the mass distribution is not spherical or asymmetric, the concept of the "length" of the pendulum and its relation to the time period of oscillation gets more involved. This experiment was designed to test the students' ability to handle the deviation from a "standard" spherical pendulum. An asymmetric object also ensured that a student cannot use simple geometrical relations to determine the centre of gravity.

A *simple pendulum* consists of a point mass $m$ (generally spherical) suspended from a string of negligible mass and length $l$. Its other end is fixed to a rigid support O. For small displacements from the equilibrium position (shown in the Figure 1), the point mass $m$ executes simple harmonic motion with time period, $T$ (time taken for one oscillation):

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where $g$ is the acceleration due to gravity.

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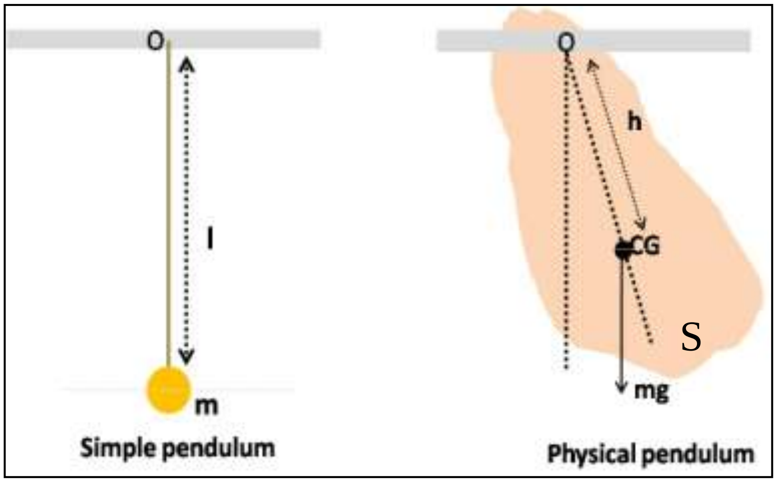



Fig. 1.

Oscillations or periodic motions pervade our Universe, but in most cases the mass distribution is aspherical and asymmetric. Therefore it would be desirable to extend the concept of length of the pendulum and its relation to the time period to a body of any arbitrary shape and size, performing simple harmonic motion. Such a pendulum is known as *compound pendulum* or *physical pendulum*.

A rigid body (one that cannot be deformed) pivoted at O (called the “point of suspension”, as shown in the above figure), or a compound pendulum, when displaced slightly executes simple harmonic motion, with time period

$$T = 2\pi \sqrt{\frac{I_0}{mgh}}$$

Here $I_0$ is the *moment of inertia* about an axis passing through the point of suspension (O), and $h$ is the distance of the point of suspension from the *centre of gravity* (CG)**.**

Moment of inertia ($I_0$) is a quantity measuring the resistance offered by a body against its rotational motion. It is always referred to with respect to an axis of rotation and it depends on the body’s shape. For a point mass $m$**,** the moment of inertia $I_0$ is given by $I_0 = mr^2$, where $r$ is the distance of the point mass from the axis of rotation.

In this experiment we consider a triangular plate of mass $m$ **(shown in pendulum assembly)**, which oscillates in its own plane. Its moment of inertia about an axis passing through its point of suspension O is given by:

$$I_0 = m(K^2 + h^2)$$

where $K$ is called the radius of gyration.

The time period of oscillation of the physical pendulum is therefore

$$T = 2\pi\sqrt{\frac{K^2 + h^2}{gh}}$$

The time period can also be written as $T = 2\pi\sqrt{\frac{L}{g}}$

where $L = \frac{K^2}{h} + h$ is called the length of an equivalent simple pendulum.

A point S (shown in Figure 1), on the other side of the CG and at a distance of $h' = \frac{K^2}{h}$ from the CG (along the line joining O and CG) is called the "point of oscillation". The oscillations with the point of suspension O are then equivalent to having all the mass concentrated at S.

**You are supplied with the following:**

| | **Quantity** |
|---|---|
| Clamp Stand | 1 |
| Triangular plate | 1 |
| Fulcrum rod with knife edge for suspension | 1 |
| Plumb line | 1 |
| Ruler | 1 |
| Stop watch | 1 |

A1 To determine the centre of gravity (CG) of triangular plate, A.

**Procedure:**

1. Suspend the triangular plate **A** (aluminum/wood/plastic) from the fulcrum rod (mounted on the clamp stand) by one of the three holes provided at the three corners of the triangle (see Figure 2.).

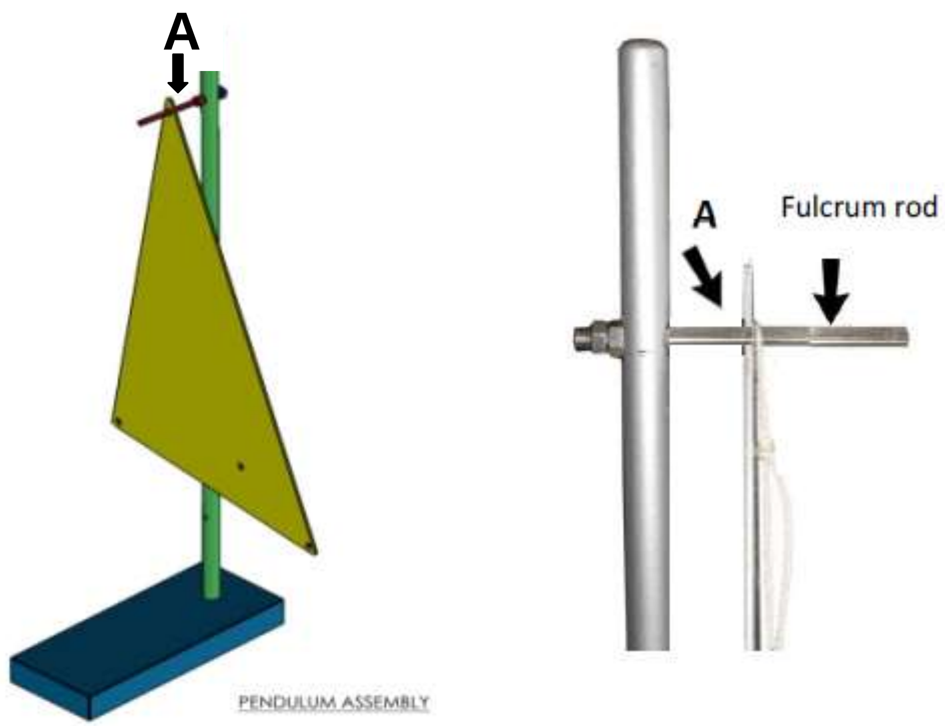



Figure 2.

2. Ensure that the suspended plate is stationary. Pass the loop of the string on the rod and hang the plumb line through the fulcrum rod (as shown in the figure above). Using a ruler and pencil, mark a straight line on the plate along the string.

3. Repeat the same procedure by suspending the plate through a different hole. The intersection of the two lines gives the CG. Use a pencil to mark it as 'X' on the plate.

   Mark the two lines and the point 'X' also on the large sized sheet of paper (provided to you) with a drawing of the triangular plate on it. Label it as **Sheet 1**.

**[A.Q1: 1.0 mark]**

4. Suspend the plate through a different hole and repeat steps 1 and 2. This line should also pass through the CG. Show the line on **Sheet 1** also.

   ***Note: The correct determination of CG is very important as any error here will introduce a corresponding error in the measurement of h, which will be used later.***

A2 To record the time period of oscillation for different suspension points for the plate.

**Procedure:**

1. Suspend the plate from hole **H1** using the fulcrum rod. Ensure that the plate is almost at the centre of the fulcrum rod and resting on the knife edge (see the figure below). This is important to reduce damping of the oscillations and, hence, to minimize error in determining the time period of oscillations.

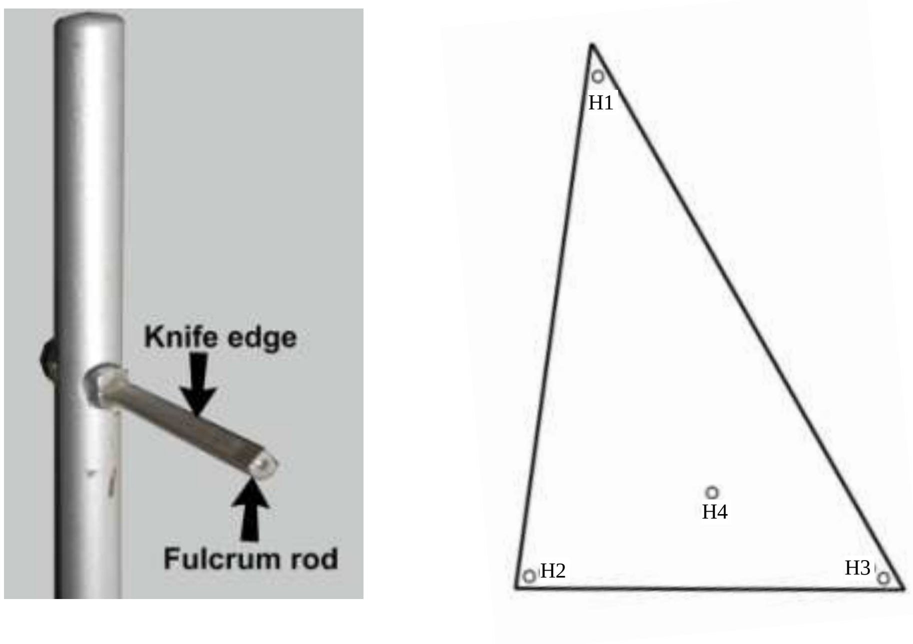



Figure 3.

**Note: Measure all distances from the top end of all the holes**. The holes are indicated in Fig. 3. [Teachers should monitor that the student does not disregard this instruction.]

2. Measure the distance **h** between hole **H1** and the **CG** you marked in the previous part of the experiment. (Measure the distance from the top end of hole **H1**). Write it in **Table A.1**.

3. Set the plate into oscillation (with small amplitude) and ensure that these oscillations occur mostly in the plane of the plate.

   [It is very IMPORTANT for teachers to note that students generally tend to make three mistakes: (i) They do not keep the pendulum very close to the bushing, near the stand. This gives enough room for pendulum to wobble due to any stray or accidental perpendicular force present. (ii) They ignore the fact to keep the amplitude small, which is essential to get accurate results and (iii) Do not rest the plate on the knife edge to reduce friction induced oscillation damping]

4. Using the stop watch, measure the time taken for 50 oscillations. Repeat three times and write each reading in **Table A.1.**

5. Repeat the above steps for holes **H2**, **H3**, and **H4.**

**[A.Q2: 4.0 marks]**

A3 To analyze the above data and determine

a) the acceleration due to gravity as an confirmatory test for the harmonic motion
b) the radius of gyration of the plate about an axis passing through its CG normal to the plane of the triangle;
c) the positions of the corresponding points of oscillation from the CG for two points of suspension; and
d) the lengths of the equivalent simple pendulum for these two points of suspension.

**Procedure:**

1. Using the data in **Table A.1**, plot a graph of $hT^2$(y-axis in ms$^2$) versus $h^2$ (x-axis in m$^2$) on the grid provided in the answer sheet (**Grid 1**).

**[A.Q3: 2.0 marks]**

2. Draw a straight line through the points (best fit) and determine the slope $s$ and the y-intercept $c$.

   Using these values of $s$ and $c$, and the expression for the time period of a physical pendulum, determine the values of $g$ in ms$^{-2}$ and $K$ in units of meters. Enter the values of $s$, $c$, $g$, and $K$ in **Table A.2.**

**[A.Q4: 3.0 marks]**

[Teachers should not give the final expression directly to students. It should be given as a derivation exercise. It will give them the insight on how to convert non-linear relations into linear relations. The equation is

$$T = 2\pi\sqrt{\frac{K^2 + h^2}{gh}}$$

$$T = 2\pi\sqrt{\frac{K^2 + h^2}{gh}}$$

$$\frac{ghT^2}{4\pi^2} = K^2 + h^2$$

$$hT^2 = \frac{4\pi^2}{g}h^2 + \frac{4\pi^2 K^2}{g}$$

The equation is -of the straight line or having the form y= mx + c]

3. For holes **H1** and **H4**, calculate the positions of the corresponding points of oscillation from the CG (h'). Write it in **Table A.3**. On the large sized sheet of paper (**Sheet 1**), mark the

positions of the points of oscillation **J1** and **J4** corresponding to the holes **H1** and **H4, respectively.**

**[A.Q5: 3.0 marks]**

4. Determine the length ($L$) of the equivalent simple pendulum when the plate is suspended from **H1** and **H4**. Write your answer in **Table A.4.**

**[A.Q6: 1.0 mark]**

In the 2013 Olympiad 226 students carried out this experiment (in groups of 3). The total marks of this experiment were 14. The distribution of the marks received by students can be seen in Figure 4.

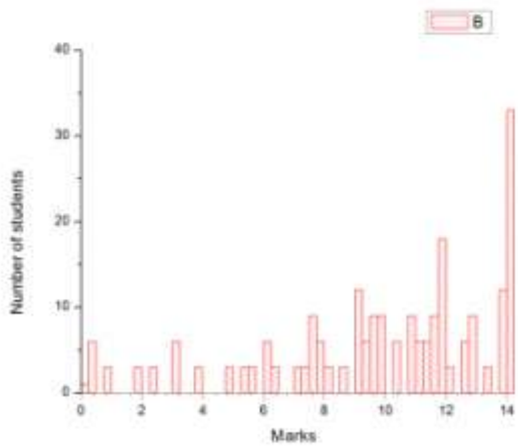



Fig. 4

Here it can be seen that 45 students could carry out the procedure flawlessly as they got ≥ 13.75 marks out of 14.

It is also interesting to try out different shapes to verify the validity of the formulation. Two of such shapes are shown below (though not tested). Instead of metal plates, hard cardboard sheets can also be used for the experiment. Depending on the geometry, the *CG* can actually be outside the plate. The following shapes are particularly useful for demonstrating that *CG* need not be within the plate.

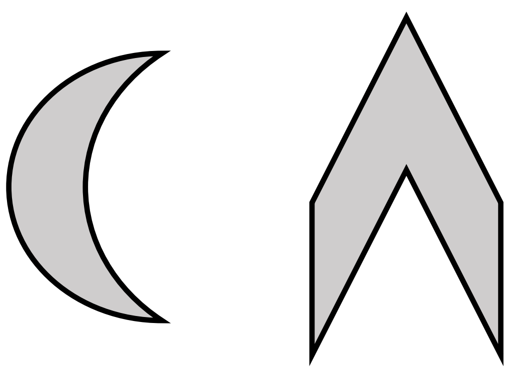


Fig. 5

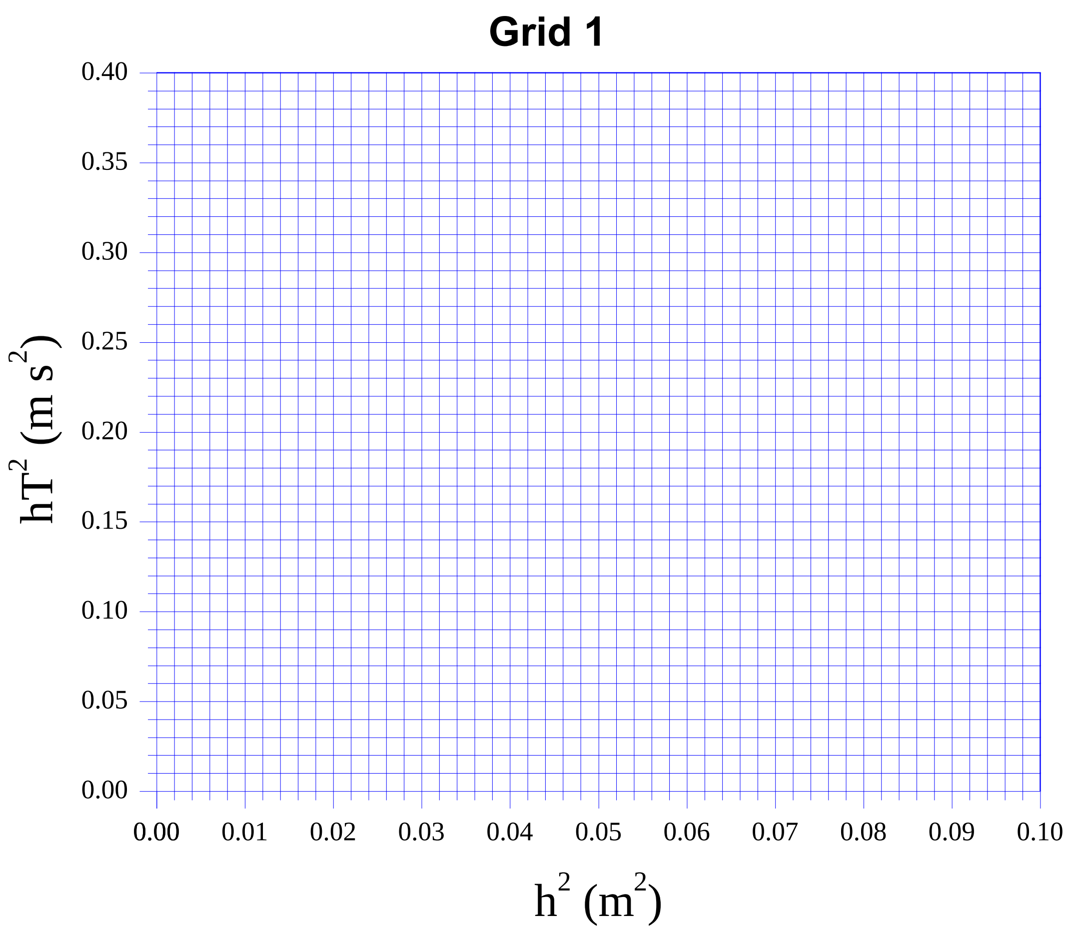
[Just as framing a question is important, designing an appropriate answer sheet is equally important.]

**Task** : This task is divided into three parts. **Total marks: 14**

**A1:** To determine the centre of gravity of a triangular plate, A.
**A2:** To record the time period of oscillation for different suspension points for the plate.
**A3:** To analyze the above data and results.

**A.Q1 Determination of CG:** **[1.0 mark]**

Mark "X" on **Sheet 1** at the appropriate position to denote the CG (large sized sheet).



**A.Q2 Table A.1: Oscillation measurements:** **[4.0 marks]**

| | h (m) | $h^2$ ($m^2$) | Time taken for 50 oscillations (s) | | | | T = T1/50 (s) | $T^2$ ($s^2$) | $hT^2$ ($ms^2$) |
|---|---|---|---|---|---|---|---|---|---|
| | | | $1^{st}$ (t1) | $2^{nd}$ (t2) | $3^{rd}$ (t3) | Mean (T1) (t1+t2+t3)/3 | | | |
| H1 | | | | | | | | | |
| H2 | | | | | | | | | |
| H3 | | | | | | | | | |
| H4 | | | | | | | | | |

**A.Q3 Results of the data analysis**

**(a) Grid 1: $hT^2$ (y-axis) versus $h^2$ (x-axis)** **[2.0 marks]**

**[ Teachers can use a regular graph paper in place of the grid in answer 3.]**

**A.Q4 Table A.2: Calculations from Grid 1** **[3.0 marks]**

| Quantity | Numerical value | Unit |
|---|---|---|
| Slope of the graph (*s*) | | |
| y-intercept of the graph (*c*) | | |
| Acceleration due to gravity (*g*) | | |
| Radius of gyration (*K*) | | |

**A.Q5 (a) Table A.3:**

**[3.0 marks]**

| Holes | h (m) | h' (m) |
|---|---|---|
| **H1** | | |
| **H4** | | |

**(b) Sheet 1**: Mark the positions of points of oscillation J1 and J4 on **Sheet 1**. Label them as J1 and J4 clearly.

**A.Q6 Table A.4: Lengths of equivalent simple pendulums** **[1.0 mark]**

| Holes | h (m) | L (m) |
|---|---|---|
| **H1** | | |
| **H4** | | |