1. Prove that Boltzmann’s H-function for a classical dilute gas has a minimum value when the distribution $f(v, t)$ is the Maxwell-Boltzmann distribution, subject to the constraints that

$$
\int d^3v f(v, t) = N/V, \quad \int d^3v f(v, t) mv^2/2 = N\epsilon/V. \tag{1}
$$

Is the assumption of ‘molecular chaos’ necessary for the proof?

2. (i) Prove that at equilibrium, Boltzmann’s H-function is equal to $-S/kV$, where $S$ is the entropy of the classical ideal gas.

(ii) Prove that if the quantum mechanical $H$-function is defined as $\Sigma P_r \ln P_r$, where $P_r$ is the probability, according to the canonical distribution, for the system to be in a state with energy $E_r$, then the entropy $S$ is related to $H$ as $S = -kH$.

3. (i) Consider the situation where a system is in equilibrium with a heat reservoir but does not have a fixed number of particles. It can exchange particles with the reservoir in such a way that the total number $N_0$ of particles in the system and the reservoir is constant. Using methods similar to those developed in the class while deriving the canonical distribution, prove that the probability for the system to be in a state with energy $E_r$ and $N_r$ particles is

$$
P_r \propto e^{-\beta E_r - \alpha N_r}, \tag{2}
$$

where $\alpha \equiv [\partial \ln \Omega'/\partial N']$ gives the variation of the number of accessible states of the reservoir w.r.t. the number of particles $N'$ in the reservoir, and is to be evaluated at $N' = N_0$.

(ii) Now consider a gas of weakly interacting bosons maintained at a temperature $T$. The total number of particles is not specified exactly but only the mean total number $\overline{N}$ is given. Use the result in Part (i) above to calculate the mean number of particles $\overline{n_s}$ in a single particle state $s$. 

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4. A set of \( N \) classical oscillators in one dimension is given by the Hamiltonian

\[
H = \sum_i \left( \frac{p_i^2}{2m} + \frac{1}{2} m \omega_i^2 q_i^2 \right)
\]  

Using the formalism of canonical ensemble in classical phase space, obtain expressions for the partition function, the energy per oscillator, the entropy per oscillator and the specific heat. Compare with the classical limit of the quantum oscillator. Calculate the expression for mean square deviation of energy as a function of \( T \).

5. Consider non-interacting particles subjected to a harmonic potential. Calculate the partition function for
   a) a single particle
   b) two distinguishable particles
   c) two spinless fermions
   d) two spin zero bosons
   e) two spin 1/2 fermions
   Compare the internal energies and entropies in these various cases. Study the limits
   \[
   T \to 0, \quad T \to \infty, \quad \hbar \to 0.
   \]

6. Reif Book II. Problem 6.7 [Spin of nuclei in a crystalline solid]

7. Reif Book II. Problem 7.11 [Specific heat of graphite]

8. Reif Book II. Problem 9.4 [Chemical potential of an ideal gas]

9. Reif Book II. Problem 9.18 [Ideal Fermi gas at \( T=0 \)]

10. Reif Book II. Problem 9.20 [Conduction electrons in Sodium]